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Modeling and forecasting dynamic conditional correlations with opening, high, low, and closing prices

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ABSTRACT

Models for variances and covariances of asset returns are crucial in risk management and asset allocation. Traditionally, these models were based on daily returns. Daily opening, high, low and closing (OHLC) prices have been sometimes used in multivariate volatility models for variances, but not for correlations. We therefore suggest a new version of the Dynamic Conditional Correlation (DCC) model wherein information from daily OHLC prices is utilized in both variance and correlation equations. The model is evaluated for two datasets: five exchange traded funds and five currencies. The results show that in terms of conditional covariance matrix estimates and forecasts the proposed model significantly outperforms, not only the standard DCC model, but also models that incorporate OHLC prices only in the variance equation.

1. Introduction

Risk assessment is an integral part of practically all decision-making. In finance, the most important source of risk is uncertainty about asset return. Estimation and forecasting of time-varying covariances of asset returns play a crucial role in asset allocation and risk management (Carroll et al., 2017; Engle et al., 2019; Harris et al., 2017). Most volatility models applied in finance are based only on closing prices. Meanwhile, daily opening, high and low prices are easily available together with daily closing prices for most financial assets. These prices can be used to construct precise estimates of volatility (Parkinson, 1980; Garman and Klass, 1980; Molnár, 2012), and these estimates have been used in several studies, e.g. Mixon (2007), Karanasos and Kartsaklas (2009) and Bjursell et al. (2010).

Opening, high, low and closing (OHLC) prices have been particularly useful in the construction of volatility models, where they contribute to more accurate estimates and forecasts of variances (see e.g. Chou, 2005; Chen et al., 2008; Hung et al., 2013; Fiszeder and Perczak, 2016; Molnár, 2016; Wu and Hou, 2020) and value-at-risk measures (see, e.g. Fiszeder et al., 2019; Meng and Taylor, 2020). Moreover, volatility forecasts from models based on daily OHLC prices are as good as volatility forecasts from models based on high-frequency data, except for short-term forecasts (Lyócsa et al., 2021). The application of such prices also has significant economic consequences (Chou and Liu, 2010; Wu and Liang, 2011). A short review of range-based models can be found in Petropoulos et al. (2022).

OHLC prices can also be applied to the multivariate volatility models. They can be used in two different ways. In the first approach, range, i.e., the difference between high and low prices, is used directly for volatility modeling. This group of applications is very wide and contains many different parameterizations of models like: range-based multivariate stochastic volatility (Tims and Mahieu, 2006), range-based DCC (Chou et al., 2009), double smooth transition conditional correlation CARR

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(Chou and Cai, 2009), multivariate range-based volatility models with dynamic copulas (Chiang and Wang, 2011; Wu and Liang, 2011), return and range heterogeneous general asymmetric DCC (Asai, 2013), range-based Markov-switching DCC (Su and Wu, 2014), DCC-range-GARCH (Fiszeder et al., 2019), IDR-DCC-NL (De Nard et al., 2021). Most of these multivariate models are constructed based on the univariate range-based volatility models. In the second approach, ranges are used not only to model volatility directly but also relations between assets. This group is very narrow and contains only three models: multivariate CARR (Fernandes et al., 2005), BEKK-HL (Fiszeder, 2018), co-range DCC (Fiszeder and Fałdziński, 2019). In all these models the estimator of covariance is based on the transformed formula for the variance of the sum of two random variables (this idea was suggested by Brunetti and Lildholdt (2002) and Brandt and Diebold (2006) but they did not apply it in any multivariate model). Models which belong to the first group also describe dependence between financial instruments but do not use estimators of covariance based on OHLC prices. For this reason, information about OHLC prices is used comprehensively only in the second approach.

Unfortunately, the possibilities of the usage of the existing volatility models with estimators of covariance based on OHLC prices are very limited because they require that the range of a portfolio return is given. It can be calculated only in some particular cases, for example, when cross rates of foreign exchange rates are given or when tick-by-tick data are available (see Fiszeder and Fałdziński, 2019).

This study has three main contributions. The first one is a proposition of the DCC model based on OHLC prices (denoted by DCC-OHLC). We use the range-based univariate volatility model range-GARCH (RGARCH) of Molnár (2016) instead of the GARCH model used in the first stage of estimation to describe conditional variances. However, at the same time, the correlation estimator of Popov (2016) based on OHLC prices is applied in the dynamic conditional correlation (DCC) model (Engle, 2002; Tse and Tsui, 2002) in the second stage of estimation. It means that we use OHLC prices directly, not only for the estimation of variances, but also for correlations of returns. To the best of our knowledge, the proposed model is the first multivariate volatility model with the correlation estimator based on OHLC prices that can be applied to any assets for which such daily prices are available.

The second contribution is to demonstrate that the use of OHLC prices in the formulation of the DCC model can improve the estimation of the covariance matrix of returns, which leads to increased accuracy of the covariance matrix forecasts, compared with the standard DCC model based on closing prices.

Third, we show that covariance matrix forecasts based on the proposed model are more accurate than those obtained from two competing range-based multivariate models, i.e., the range-based DCC model of Chou et al. (2009) and the DCC-RGARCH model of Fiszeder et al. (2019). The superiority of the DCC-OHLC model results from the application of the estimator of correlation based on OHLC prices in the second stage of estimation.

The rest of the paper is organized in the following way. Section 2 provides a description of the applied models. In Section 3 the performance of the model is compared with the three other DCC models: standard DCC, range-based DCC and DCC-RGARCH for five selected exchange-traded funds: SPDR Portfolio S&P 500 Growth, iShares Core U.S. Aggregate Bond, iShares U.S. Real Estate, United States Oil Fund, SPDR Gold Shares and five exchange rates: euro (EUR), Japanese yen (JPY), British pound (GBP), Australian dollar (AUD), Canadian dollar (CAD) against the U.S. dollar (USD). Section 4 provides conclusions.

2. Theoretical background

In the paper, we introduce the DCC-OHLC model and compare it with three competing multivariate GARCH models: the DCC model of Engle (2002), the range-based DCC model of Chou et al. (2009) and the DCC-RGARCH model of Fiszeder et al. (2019). All three competing models are very similar in their correlation part but differ in their specification for univariate conditional variances. The first DCC model is based on the GARCH model of Bollerslev (1986), the range-based DCC model is formulated on the CARR model of Chou (2005) while the DCC-RGARCH model is based on the RGARCH model of Molnár (2016). In the following sections, we present the Popov correlation estimator and the competing DCC models.

2.1. Popov correlation estimator

Popov (2016) introduced the correlation estimator based on OHLC prices as a modification of the Rogers–Zhou estimator (Rogers and Zhou, 2008). It is based on the concept of the balanced excess return, which is the difference of upper and lower wicks in the Japanese candlestick representation of OHLC prices (see e.g. Nison, 1994).

Following Popov (2016) we assume without loss of generality that the opening price is normalized to one, i.e., the logarithm of the price is zero. Let H_{it}, L_{it}, C_{it} ($i = 1, 2$ and means the number of the series) be the logarithm of daily high, low and closing prices respectively. Due to normalization, H_{it}, L_{it}, C_{it} can be treated as daily high, low and closing logarithmic returns calculated relative to the opening price. Upper and lower wicks of a candlestick can be written respectively as:

$$w_{u,it} = H_{it} - \max(0, C_{it}) \text{ and } w_{l,it} = -(L_{it} - \min(0, C_{it})). \tag{1}$$

The balanced excess return is given as:

$$W_{it} = w_{u,it} - w_{l,it} = H_{it} - \max(0, C_{it}) + L_{it} - \min(0, C_{it}) = H_{it} + L_{it} - C_{it}. \tag{2}$$

The correlation estimator of Popov (2016) can be presented as:

$$\hat{\rho}_P = 0.5\left(\hat{\rho}_{\mu C} + 1.1958\hat{\rho}_{\mu W} - 0.1958\hat{\rho}^3_{\mu W}\right), \tag{3}$$

where $\hat{\rho}_{\mu C} = \frac{\sum_{t=1}^{no} C_{1t}C_{2t}}{\sqrt{\sum_{t=1}^{no} C_{1t}^2 C_{2t}^2}}$, $\hat{\rho}_{\mu W} = \frac{\sum_{t=1}^{no} W_{1t}W_{2t}}{\sqrt{\sum_{t=1}^{no} W_{1t}^2 \sum_{t=1}^{no} W_{2t}^2}}$, the subscript μ in the coefficients $\hat{\rho}_{\mu C}$ and $\hat{\rho}_{\mu W}$ indicates that the estimator is based on the assumed zero means, no is the number of observations used to estimate the coefficient.

Due to the approximation, the Popov estimator is not consistent, however, it is a minor issue. Firstly, for small to medium-sized samples, the contribution of the approximation error is negligible. The difference between the true value and the approximation, evaluated numerically on a fine grid, is not larger than 0.001554991 in absolute value. Secondly, a consistent estimator can be obtained using the true correction function. Such estimator is given as:

$$\hat{\rho}_{Pg} = 0.5\left[\hat{\rho}_C + g^{-1}(\hat{\rho}_{\mu W})\right], \tag{4}$$

where $g(\rho) = \frac{1}{3-4\ln 2}[2f(\rho) - 2f(-\rho) - \rho]$, $f(\rho) = \cos\alpha \int_0^\infty \frac{\cosh \nu\alpha}{\sinh \nu\pi/2} \tanh \nu\gamma d\nu$, $\rho = \sin\alpha$, $\alpha\epsilon(-0.5\pi, 0.5\pi)$ and $2\gamma = \alpha + 0.5\pi$.

The function g^{-1} is not available in a closed form and its values need to be computed numerically.

Popov (2016) demonstrated that his estimator is asymptotically normal. In a simulation study, he analyzed the properties of the estimator under the framework of the bivariate standard Brownian motion and showed that for small samples it has a downside bias and for reasonably-sized samples it is nearly unbiased. This result is in line with expectations because the Popov estimator can be considered as a weighted average of two Pearson-type correlation estimators, and it is well-known that the Pearson correlation estimator is biased of order $O(1/n)$. The most important thing, however, is that the Popov estimator is significantly more efficient than the Pearson correlation estimator with efficiency gains of around 65%. It is also more efficient than the Rogers–Zhou correlation estimator, except for the vicinity of zero correlation. In this case efficiency gains are increasing with the increasing absolute value of the true value of correlation.

2.2. The DCC model

The dynamic conditional correlation model was introduced independently by Engle (2002) and Tse and Tsui (2002). The main difference between these models is the formulation of the correlation matrix. In the model of Tse and Tsui, conditional correlations are the weighted sum of past conditional correlations, whereas in the model of Engle the matrix written like the GARCH equation is later transformed to the correlation matrix. In this paper, the model of Tse and Tsui is a base for the new model because in this specification the conditional correlation matrix is described directly and the Popov correlation coefficients can be easily introduced in it. Moreover, the DCC model of Engle provides dynamic correlation estimates as a product of standardization, and not as a direct result of the equation governing the multivariate dynamics (see Caporin and McAleer, 2013). The DCC model of Tse and Tsui was also a base of the co-range DCC model (Fiszeder and Fałdziński, 2019).

The DCC(P,Q)-GARCH(p,q) model of Tse and Tsui (2002) can be given as:

$$\boldsymbol{\varepsilon}_t \big| \psi_{t-1} \sim Normal(0, \mathbf{cov}_t), \tag{5}$$

$$\boldsymbol{\Xi}_{t-1} = \mathbf{B}_{t-1}^{-1}\mathbf{S}_{t-1}\mathbf{S'}_{t-1}\mathbf{B}_{t-1}^{-1}, \tag{6}$$

$$\mathbf{cor}_t = \left(1 - \sum_{i=1}^{Q}\zeta_i - \sum_{j=1}^{P}\theta_j\right)\mathbf{cor} + \sum_{i=1}^{Q}\zeta_i \boldsymbol{\Xi}_{t-i} + \sum_{j=1}^{P}\theta_j \mathbf{cor}_{t-j}, \tag{7}$$

$$\mathbf{cov}_t = \mathbf{D}_t \mathbf{cor}_t \mathbf{D}_t, \tag{8}$$

where $\boldsymbol{\varepsilon}_t$ is the N-dimensional innovation process for the conditional mean, ψ_{t-1} is the set of all information available at time $t-1$, $Normal$ is the conditional multivariate normal distribution, $\mathbf{cov}_t$ is the $N \times N$ symmetric conditional covariance matrix, $\boldsymbol{\Xi}_{t-1}$ is the $N \times N$ sample estimate of the conditional correlation matrix based on recent M standardized residuals $\{\mathbf{z}_{t-1}, \mathbf{z}_{t-2}, \ldots, \mathbf{z}_{t-M}\}$, $\mathbf{z}_t$ is the standardized $N \times 1$ residual vector assumed to be serially independently distributed given as $\mathbf{z}_t = \mathbf{D}_t^{-1}\boldsymbol{\varepsilon}_t$, $\mathbf{D}_t = \mathrm{diag}(h_{1t}^{0.5}, h_{2t}^{0.5}, \ldots, h_{Nt}^{0.5})$, conditional variances h_{kt} (for $k = 1, 2, \ldots, N$) are described as univariate GARCH models, $\mathbf{B}_{t-1}$ is the $N \times N$ diagonal matrix with the kth diagonal element being $\left(\sum_{h=1}^{M} z_{kt-h}^2\right)^{0.5}$, $z_{kt} = \varepsilon_{kt}/\sqrt{h_{kt}}$, $\mathbf{S}_{t-1}$ is the $N \times M$ matrix given as $\mathbf{S}_{t-1} = (\mathbf{z}_{t-1}, \ldots, \mathbf{z}_{t-M})$, $\mathbf{cor}_t$ is the conditional $N \times N$ correlation matrix of $\boldsymbol{\varepsilon}_t$, $\mathbf{cor}$ is the unconditional sample $N \times N$ correlation matrix of $\boldsymbol{\varepsilon}_t$ (the sample size is n). The parameters ζ_i (for $i = 1, 2, \ldots, Q$), θ_j (for $j = 1, 2, \ldots, P$) are nonnegative and satisfy the condition $\sum_{i=1}^{Q}\zeta_i + \sum_{j=1}^{P}\theta_j < 1$.

The univariate GARCH(p,q) model (introduced by Bollerslev, 1986), applied in the DCC-GARCH model, can be written as:

$$\varepsilon_{kt} \big| \psi_{t-1} \sim Normal(0, h_{kt}), \qquad k = 1, 2, \ldots, N, \tag{9}$$

$$h_{kt} = \alpha_{k0} + \sum_{i=1}^{q}\alpha_{ki}\varepsilon_{kt-i}^2 + \sum_{j=1}^{p}\beta_{kj}h_{kt-j}, \tag{10}$$

where ε_{kt} is the univariate innovation process for the conditional mean, $Normal$ is the conditional normal distribution, $\alpha_{k0} > 0, \alpha_{ki} \geq 0, \beta_{kj} \geq 0$ (for $k = 1, 2, \ldots, N;\ i = 1, 2, \ldots, q;\ j = 1, 2, \ldots, p$), however, weaker conditions for non-negativity of the conditional variance can be assumed (see Nelson and Cao, 1992). The requirement for covariance stationarity of ε_{kt} is $\sum_{i=1}^{q}\alpha_{ki} + \sum_{j=1}^{p}\beta_{kj} < 1$.

Denoting $\Xi_t = \{\xi_{ijt}\}$, the ijth element of Ξ_{t-1} is given as:

$$\xi_{ijt-1} = \frac{\sum_{h=1}^{M} z_{it-h} z_{jt-h}}{\sqrt{\left(\sum_{h=1}^{M} z_{it-h}^2\right)\left(\sum_{h=1}^{M} z_{jt-h}^2\right)}}. \tag{11}$$

The positive definiteness of $\mathbf{cor}_t$ is ensured by construction if $\mathbf{cor}_t$ and Ξ_t (for $t \leq 0$) and Ξ_{t-i} (for $t > 0$) are positive definite. A necessary condition for the latter to hold is $M \geq N$.

The parameters of the DCC-GARCH model can be estimated by the QML method using a two-stage approach. The log-likelihood function can be written as:

$$L(\boldsymbol{\Theta}) = L_{Vol}(\boldsymbol{\Theta}_1) + L_{Corr}(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1), \tag{12}$$

where $\boldsymbol{\Theta}' = (\boldsymbol{\Theta}'_1, \boldsymbol{\Theta}'_2)$, $\boldsymbol{\Theta}_1$ is the vector of the parameters of conditional means and variances and $\boldsymbol{\Theta}_2$ is the vector of the parameters of the correlation part of the model, $L_{Vol}(\boldsymbol{\Theta}_1)$ represents the volatility part:

$$L_{Vol}(\boldsymbol{\Theta}_1) = -0.5 \sum_{k=1}^{N} \left(n \ln(2\pi) + \sum_{t=1}^{n} \left(ln(h_{kt}) + \frac{\epsilon_{kt}^2}{h_{kt}} \right) \right), \tag{13}$$

while $L_{Corr}(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1)$ can be viewed as the correlation component:

$$L_{Corr}(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1) = -0.5 \sum_{t=1}^{n} \left(\ln |\mathbf{cor}_t| + \mathbf{z}'_t \mathbf{cor}_t^{-1} \mathbf{z}_t - \mathbf{z}'_t \mathbf{z}_t \right), \tag{14}$$

where n is the number of observations used in estimation.

In the first stage, the parameters of univariate GARCH models can be estimated separately for each of the assets (the function is conditional on pre-sample estimates of h_{kt} and ϵ_{kt}^2, for $t \leq 0$; for h_{kt} the sample variance of the observed data can be used and $\varepsilon_{kt} = 0$ can be assumed) and in the second stage residuals transformed by their estimated standard deviations are applied to estimate the parameters of the correlation part ($\boldsymbol{\Theta}_2$) conditioning on the parameters estimated in the first stage ($\widehat{\boldsymbol{\Theta}}_1$) and matrices $\mathbf{cor}_t$ and $\boldsymbol{\Xi}_t$ for $t \leq 0$ (as $\mathbf{cor}_t$ the sample unconditional correlation matrix $\mathbf{S}$ can be applied and for $\boldsymbol{\Xi}_t$ zero matrix; these assumptions have no effects on the asymptotic distribution of the QML estimator).

2.3. The DCC-CARR model

In this paper, the new DCC-OHLC model is compared not only with the DCC-GARCH model, formulated on closing prices, but also with the range-based DCC model (introduced by Chou et al., 2009) which is formulated using low and high prices. We refer to it as the DCC-CARR model in this paper because it is based on the CARR model. The DCC(P, Q)-CARR(p, q) model can be expressed as

$$\boldsymbol{\varepsilon}_t | \psi_{t-1} \sim Normal(0, \mathbf{cov}_t), \tag{15}$$

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^{Q} \zeta_i - \sum_{j=1}^{P} \theta_j \right) \mathbf{S} + \sum_{i=1}^{Q} \zeta_i (\mathbf{z}_{t-i}^{CARR} (\mathbf{z}_{t-i}^{CARR})') + \sum_{j=1}^{P} \theta_j \mathbf{Q}_{t-j}, \tag{16}$$

$$\mathbf{cor}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1}, \tag{17}$$

$$\mathbf{cov}_t = \mathbf{D}_t \mathbf{cor}_t \mathbf{D}_t, \tag{18}$$

where $\mathbf{z}_t^{CARR}$ is the standardized $N \times 1$ residual vector which contains the standardized residuals z_{kt}^{CARR} calculated from the CARR model (Eqs. (19)–(21)) as $z_{kt}^{\mathrm{CARR}} = \varepsilon_{kt} / \lambda_{kt}^*$, $\lambda_{kt}^* = \mathrm{adj}_k \lambda_{kt}$ for $k = 1, 2, \ldots, N$, where $\mathrm{adj}_k = \frac{\bar{\sigma}_k}{\bar{\lambda}_k}$, $\bar{\sigma}_k$ is the unconditional standard deviation of returns, $\bar{\lambda}_k$ is the sample mean of the conditional range, $\mathbf{S}$ is the unconditional $N \times N$ covariance matrix of $\mathbf{z}_t^{CARR}$ (the sample size is n). $\mathbf{Q}_t^*$ is the diagonal $N \times N$ matrix composed of the square root of the diagonal elements of $\mathbf{Q}_t$, $\mathbf{D}_t = \mathrm{diag}\left(\lambda_{1t}^*, \lambda_{2t}^*, \ldots, \lambda_{Nt}^*\right)$, the other variables are defined in the same way as in the DCC-GARCH model.

The CARR(p, q) model (introduced by Chou, 2005), applied in the DCC-CARR model, can be described as:

$$u_{kt} | \psi_{t-1} \sim \exp\left(1, \xi_t\right), \qquad k = 1, 2, \ldots, N, \tag{19}$$

$$R_{kt} = \lambda_{kt} u_{kt}, \tag{20}$$

$$\lambda_{kt} = \alpha_{k0} + \sum_{i=1}^{q} \alpha_{ki} R_{kt-i} + \sum_{j=1}^{p} \beta_{kj} \lambda_{kt-j}, \tag{21}$$

where R_{kt} is the price range given as $R_{kt} = ln\left(H_{kt}\right) - ln\left(L_{kt}\right)$, H_{kt} and L_{kt} are high and low prices over a fixed period (in our study during a day), λ_{kt} is the conditional mean of the range and u_{kt} is the disturbance term. The exponential distribution is a natural choice for the conditional distribution of u_{kt} because it takes positive values. For positivity of λ_{kt} and weakly stationary, similar conditions like in the GARCH model have to be imposed.

The parameters of the DCC-CARR model can be estimated by the quasi-maximum likelihood method using a two-stage approach. The log-likelihood function can be written as the sum of two parts, the volatility part and the correlation part:

$$L^{DCC-CARR}(\boldsymbol{\Theta}) = L_{Vol}^{DCC-CARR}(\boldsymbol{\Theta}_1) + L_{Corr}^{DCC-CARR}(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1), \tag{22}$$

$$L_{Vol}^{DCC-CARR}(\boldsymbol{\Theta}_1) = -\frac{1}{2}\sum_{k=1}^{N}\left(n\ln(2\pi) + \sum_{t=1}^{n}\left(2ln(\lambda_{kt}^*) + \frac{\epsilon_{kt}^2}{\lambda_{kt}^{*2}}\right)\right) \tag{23}$$

$$L_{Corr}^{DCC-CARR}(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1) = -\frac{1}{2}\sum_{t=1}^{n}\left(\ln|\mathbf{cor}_t| + (z_t^{CARR})'\mathbf{cor}_t^{-1}z_t^{CARR} - (z_t^{CARR})'z_t^{CARR}\right). \tag{24}$$

This means that in the first stage the parameters of the CARR models can be estimated separately for each of the assets (the function is conditional on pre-sample estimates of λ_{kt} and R_{kt}, for $t \leq 0$; for λ_{kt} the sample mean of the conditional range can be used and $R_{kt} = 0$ can be assumed). The CARR model is based on a price range and describes the dynamics of the conditional mean of a price range. That is why to estimate values of the conditional standard deviation of returns, the conditional price range has to be scaled. In the second stage the standardized residuals z_{kt}^{CARR} are used to maximize Eq. (24) to estimate the parameters of the correlation component conditioning on the parameters estimated in the first stage and matrices $\mathbf{Q}_t$ and $\mathbf{z}_t^{CARR}(\mathbf{z}_t^{CARR})'$ for $t \leq 0$ (as $\mathbf{Q}_t$ the sample unconditional correlation matrix $\mathbf{S}$ can be applied and for $\mathbf{z}_t^{CARR}(\mathbf{z}_t^{CARR})'$ zero matrix).

2.4. The DCC-range-GARCH model

The third benchmark to compare with our new model is the DCC-Range-GARCH model (denoted by DCC-RGARCH) introduced by Fiszeder et al. (2019). The DCC(P, Q)-RGARCH(p, q) model can be presented as:

$$\boldsymbol{\varepsilon}_t | \psi_{t-1} \sim Normal(0, \mathbf{cov}_t), \tag{25}$$

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^{Q}\zeta_i - \sum_{j=1}^{P}\theta_j\right)\mathbf{S} + \sum_{i=1}^{Q}\zeta_i(\mathbf{z}_{t-i}^{RGARCH}(\mathbf{z}_{t-i}^{RGARCH})') + \sum_{j=1}^{P}\theta_j\mathbf{Q}_{t-j}, \tag{26}$$

$$\mathbf{cor}_t = \mathbf{Q}_t^{*-1}\mathbf{Q}_t\mathbf{Q}_t^{*-1}, \tag{27}$$

$$\mathbf{cov}_t = \mathbf{D}_t\mathbf{cor}_t\mathbf{D}_t, \tag{28}$$

where $\mathbf{z}_t^{RGARCH}$ is the standardized $N \times 1$ residual vector which contains the standardized residuals z_{kt}^{RGARCH} calculated from the RGARCH model as $z_{kt}^{\mathrm{RGARCH}} = \varepsilon_{kt}/\left(h_{kt}^{RGARCH}\right)^{1/2}$, conditional variances h_{kt}^{RGARCH} (for $k = 1, 2, \dots, N$) are described as for the RGARCH model (Eqs. (29)–(30)), $\mathbf{S}$ is the unconditional $N \times N$ covariance matrix of $\mathbf{z}_t^{RGARCH}$ (the sample size is n), $\mathbf{D}_t = \mathrm{diag}\left((h_{1t}^{RGARCH})^{1/2}, (h_{2t}^{RGARCH})^{1/2} \dots, (h_{Nt}^{RGARCH})^{1/2}\right)$, the other variables are defined in the same way as in the DCC-GARCH and DCC-CARR models.

The univariate range-GARCH(p, q) (RGARCH) model (introduced by Molnár, 2016), applied in the DCC-RGARCH model, can be written as:

$$\varepsilon_{kt} | \psi_{t-1} \sim Normal(0, h_{kt}^{RGARCH}), \qquad k = 1, 2, \dots, N, \tag{29}$$

$$h_{kt}^{RGARCH} = \alpha_{k0} + \sum_{i=1}^{q}\alpha_{ki}\sigma_{k,Pt-i}^2 + \sum_{j=1}^{p}\beta_j h_{kt-j}^{RGARCH}, \tag{30}$$

where $\sigma_{k,Pt}^2$ is the Parkinson estimator of variance (Parkinson, 1980) given as $\sigma_{k,Pt}^2 = [ln(H_{kt}/L_{kt})]^2/(4\ln 2)$. For positivity of h_{kt}^{RGARCH} and weakly stationary, similar conditions like in the GARCH model have to be imposed.

The parameters of the DCC-RGARCH model can be estimated by the quasi-maximum likelihood method using a two-stage approach. The log-likelihood function can be written as the sum of two parts: the volatility part and the correlation part:

$$L^{DCC-RGARCH}(\boldsymbol{\Theta}) = L_{Vol}^{DCC-RGARCH}(\boldsymbol{\Theta}_1) + L_{Corr}^{DCC-RGARCH}(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1), \tag{31}$$

$$L_{Vol}^{DCC-RGARCH}(\boldsymbol{\Theta}_1) = -\frac{1}{2}\sum_{k=1}^{N}\left(n\ln(2\pi) + \sum_{t=1}^{n}\left(ln(h_{kt}^{RGARCH}) + \frac{\epsilon_{kt}^2}{h_{kt}^{RGARCH}}\right)\right) \tag{32}$$

$$L_{Corr}^{DCC-RGARCH}(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1) = -\frac{1}{2}\sum_{t=1}^{n}\left(\ln|\mathbf{cor}_t| + (z_t^{RGARCH})'\mathbf{cor}_t^{-1}z_t^{RGARCH} - (z_t^{RGARCH})'z_t^{RGARCH}\right), \tag{33}$$

This means that in the first stage the parameters of univariate RGARCH models can be estimated separately for each of the assets. High and low prices are used in the RGARCH model to calculate the Parkinson estimator, but at the same time, the estimation of its parameters is based on closing returns. This is because the Parkinson estimator is a scaled price range and describes the variance of returns. The function is conditional on pre-sample estimates of h_{kt}^{RGARCH} and $\sigma_{k,Pt}^2$, for $t \leq 0$. A natural choice for h_{kt}^{RGARCH} is the sample variance of the observed data and for $\sigma_{k,Pt}^2$ zero value can be taken.

In the second stage the standardized residuals z_{kt}^{RGARCH} are used to maximize Eq. (33) in order to estimate the parameters of the correlation component conditioning on the parameters estimated in the first stage and matrices $\mathbf{Q}_t$ and $\mathbf{z}_{t-i}^{RGARCH}(\mathbf{z}_{t-i}^{RGARCH})'$ for $t \leq 0$ (as $\mathbf{Q}_t$ the sample unconditional correlation matrix $\mathbf{S}$ can be applied and for $\mathbf{z}_{t-i}^{RGARCH}(\mathbf{z}_{t-i}^{RGARCH})'$ zero matrix).

2.5. DCC-OHLC model

In this section, we introduce the new formulation of the DCC model with the use of the correlation estimator based on OHLC prices. It is based on the RGARCH model (described in Section 2.4), which is applied in the first stage of estimation and the DCC model (described in Section 2.2) with the Popov correlation estimator (described in Section 2.1) in the second stage. The DCC(P,Q)-RGARCH(p,q) model based on OHLC prices (denoted by DCC-OHLC) can be written as:

$$\boldsymbol{\varepsilon}_t | \psi_{t-1} \sim Normal(0, \mathbf{cov}_t), \tag{34}$$

$$\mathbf{cor}_t = \left(1 - \sum_{i=1}^{Q} \zeta_i - \sum_{j=1}^{P} \theta_j\right) \mathbf{cor} + \sum_{i=1}^{Q} \zeta_i \boldsymbol{\Phi}_{t-i} + \sum_{j=1}^{P} \theta_j \mathbf{cor}_{t-j}, \tag{35}$$

$$\mathbf{cov}_t = \mathbf{D}_t \mathbf{cor}_t \mathbf{D}_t, \tag{36}$$

where $\mathbf{D}_t = \text{diag}\left(\left(h_{1t}^{RGARCH}\right)^{0.5}, \left(h_{2t}^{RGARCH}\right)^{0.5} \dots, \left(h_{Nt}^{RGARCH}\right)^{0.5}\right)$, the conditional variances h_{kt}^{RGARCH} ($k = 1, 2, \dots, N$) are described as the RGARCH model (Eqs. (29)–(30)), $\boldsymbol{\Phi}_t$ is the $N \times N$ conditional symmetric correlation matrix given as:

$$\boldsymbol{\Phi}_t = \begin{bmatrix} 1 & \hat{\rho}_{Pt,12} & \cdots & \hat{\rho}_{Pt,1N} \\ \hat{\rho}_{Pt,21} & 1 & \cdots & \hat{\rho}_{Pt,2N} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\rho}_{Pt,N1} & \hat{\rho}_{Pt,N2} & \cdots & 1 \end{bmatrix}, \tag{37}$$

$\hat{\rho}_{Pt,kl}$ (for $k, l = 1, 2, \dots, N$) is the Popov correlation estimator between daily returns of assets k and l given in Eq. (3). The other symbols are defined in the same way as in the DCC-GARCH model in Section 2.2.

After the preliminary tests, we estimate the Popov correlation, based on the 5 previous days ($no = 5$ in Eq. (3)). It is a compromise between the size of the underestimation of the coefficient for too few observations and too slow response to current market changes for a larger number of observations. There are two essential changes in the DCC-OHLC model in comparison to the DCC model of Tse and Tsui (2002). Firstly, instead of the univariate GARCH model, we apply the RGARCH model which is based on the range-based variance estimator. Secondly, we incorporate the correlation estimator based on OHLC prices in the calculation of the conditional correlation matrix. It is worth emphasizing that the proposed model is parsimonious, and there are no additional parameters in comparison to the standard DCC model based on returns of closing prices.

The Popov estimator is defined for open-to-close returns (see Section 2.1). The opening jump (the difference between today's opening price and yesterday's closing price) can deteriorate its properties for the close-to-close returns. For that reason, we suggest applying the DCC-OHLC model for open-to-close returns.

The parameters of the DCC-OHLC model can be estimated by the quasi-maximum likelihood method using a two-stage approach. The log-likelihood function can be written as:

$$L^{DCC-OHLC}(\boldsymbol{\Theta}) = L_{Vol}^{DCC-OHLC}\left(\boldsymbol{\Theta}_1\right) + L_{Corr}^{DCC-OHLC}\left(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1\right), \tag{38}$$

where the volatility part is expressed as:

$$L_{Vol}^{DCC-OHLC}\left(\boldsymbol{\Theta}_1\right) = -0.5 \sum_{k=1}^{N} \left(n \ln(2\pi) + \sum_{t=1}^{n} \left(ln(h_{kt}^{RGARCH}) + \frac{\epsilon_{kt}^2}{h_{kt}^{RGARCH}}\right)\right) \tag{39}$$

and the correlation component has the form:

$$L_{Corr}^{DCC-OHLC}\left(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1\right) = -0.5 \sum_{t=1}^{n} \left(\ln|\mathbf{cor}_t| + (\mathbf{z}_t^{RGARCH})' \mathbf{cor}_t^{-1} \mathbf{z}_t^{RGARCH} - (\mathbf{z}_t^{RGARCH})' \mathbf{z}_t^{RGARCH}\right), \tag{40}$$

$\mathbf{z}_t^{RGARCH}$ is the standardized $N \times 1$ residual vector which contains the standardized residuals z_{kt}^{RGARCH} calculated from the RGARCH model as $z_{kt}^{\text{RGARCH}} = \varepsilon_{kt} / \left(h_{kt}^{RGARCH}\right)^{0.5}$. It means that in the first stage the parameters of univariate RGARCH models can be estimated separately for each of the assets (the function (39) is conditional on pre-sample estimates of h_{kt}^{RGARCH} and σ_{Pt}^2, for $t \le 0$; for h_{kt}^{RGARCH} the sample variance of the observed data can be used and $\sigma_{Pt}^2 = 0$ can be assumed). In the second stage, the standardized residuals z_{kt}^{RGARCH} are used to maximize Eq. (40) in order to estimate the parameters of the correlation component ($\boldsymbol{\Theta}_2$) conditioning on the parameters estimated in the first stage ($\hat{\boldsymbol{\Theta}}_1$) and matrices $\mathbf{cor}_t$ and $\boldsymbol{\Phi}_t$ for $t \le 0$ (as $\mathbf{cor}_t$ the sample unconditional correlation matrix can be applied and for $\boldsymbol{\Phi}_t$ zero matrix).

3. Analysis of exchange-traded funds and exchange rates

We apply the proposed model and its competitors to two different sets of data: five exchange-traded funds (ETFs) and five currency rates. The analyzed ETFs are listed on the New York Stock Exchange Arca, namely: SPDR Portfolio S&P 500 Growth (holds large-capitalization growth stocks selected from the S&P 500 index), iShares Core U.S. Aggregate Bond (holds U.S. investment-grade bonds), iShares U.S. Real Estate (holds U.S. real estate companies and REITs), United States Oil Fund (holds crude oil futures contracts and other oil-related contracts, predominantly short-term NYMEX futures contracts on WTI crude oil), and SPDR Gold

Shares (holds gold bullion). The second set includes the five most heavily traded currency pairs in the Forex market, namely: EUR/USD, USD/JPY, GBP/USD, AUD/USD, and USD/CAD.

The dynamics of the opening jump (the difference between today's opening price and yesterday's closing price) is arguably different from the dynamics of the trading part of the day. As is mentioned in Section 2.5, in order to avoid the noise induced by measuring the overnight volatility and correlation, we analyze daily open-to-close returns instead of daily close-to-close returns. For the same reason, when we calculate realized variances and covariances, we omit the opening jump. It is a common approach in the realized volatility literature (see e.g. Floros et al., 2020; Reschenhofer et al., 2020; Zhang et al., 2020; Gkillas et al., 2021; Kambouroudis et al., 2021.

The percentage logarithmic returns and ranges, i.e., multiplied by 100, are used in the paper. An evaluation of the considered models is performed for daily data spanning seven years, from January 3, 2012, to December 31, 2018. For ETFs, we exclude one day connected with a flash crash on August 24, 2015. On that day the S&P 500 index opened at 1965.15 and within minutes fell to a low of 1867.01, a 5% decline. During that day the market gained back most of the loss, but toward the close of trading stocks fell again, ending the day 3.66% below the open. For currency pairs, we omit two days. The first day is the Brexit referendum which took place on June 23, 2016. The second day is a flash crash on October 7, 2016. On that day the British pound dropped more than 6% in two minutes against the US dollar. It recovered most of the losses soon afterward. Carnero et al. (2007), Catalán and Trívez (2007), Carnero et al. (2012), and Boudt et al. (2013) show that such outliers may cause biases on the usual maximum likelihood estimator of the parameters of GARCH models and the estimated volatilities. If such outliers are not treated adequately, they can lead to a considerable deterioration of the forecasting accuracy (Catalán and Trívez, 2007; Trucíos and Hotta, 2015). During such events, all the models perform very poorly, and the inclusion of such data could bias the comparison . We leave this problem for future studies.

The estimation of model parameters is done in Gauss and the calculation of the applied tests is performed in EViews and OxMetrics programs using self-written codes. The SPA and MCS tests are performed using the Hansen and Lunde (2014) package in OxMetrics.

3.1. In-sample comparison of models

We apply four[1] DCC models in the analysis:

(1) The DCC model of Engle (2002), denoted by DCC-GARCH,

(2) The range-based DCC model of Chou et al. (2009), denoted by DCC-CARR,

(3) The DCC-RGARCH model of Fiszeder et al. (2019),

(4) The proposed DCC-OHLC model.

The first model is based only on closing prices, while the remaining models are multivariate range-based models. For the second and third models, OHLC prices are used only in the first stage of estimation (i.e. for variances), while for the fourth model, these prices are applied in both stages of estimation (i.e. for variances and covariances).

First, we compare the fit of the estimated models on the whole sample of data, i.e. from January 3, 2012, to December 31, 2018. Mean equations for returns are very simple: each mean equation is a constant because in our data the sample return of any asset is not dependent on its own past returns nor on the past returns of other assets. All models are with one lag for both the volatility and correlation parts (for instance DCC(1,1)-GARCH(1,1)).

The parameters of all models are estimated using the quasi-maximum likelihood method. The results of the estimation are presented in Tables 1–2 for ETFs and exchange rates, respectively.

It is worth noting that there are significant differences in the estimates of parameters between the considered models. The estimates of the parameters α_{k1} are much higher and the estimates of the parameters β_{k1} much lower in the CARR and RGARCH models compared with the GARCH model. This empirical regularity has already been stated in the literature (see e.g. Chou et al., 2009; Wu and Liang, 2011; Su and Wu, 2014; Fiszeder and Fałdziński, 2019; Fiszeder et al., 2019). For modeling relations between assets, differences in the correlation component of the considered DCC models are more important. The estimate of the parameter ζ_1 in the DCC-OHLC model is higher, while the estimate of the parameter θ_1 is lower compared with the estimates in other models. Shock in the previous period has a stronger impact on the current covariance of returns, and thus, the proposed model with the correlation estimator based on OHLC prices has a faster response to sudden changes in the market. A slow reply to abrupt changes is one of the greatest deficiencies of GARCH-type models (e.g. Andersen et al., 2003; Hansen et al., 2012).

As mentioned in Section 2.3, the estimation of the parameters of the CARR model is based on a price range, which is why it is not possible to compare the values of the likelihood function between the CARR model and the GARCH and RGARCH models directly. However, for the DCC-CARR it is possible to calculate the likelihood function based on the scaled conditional price range. Henceforth, it is possible to evaluate all the DCC models based on the whole likelihood function including volatility and correlation parts. We apply the Rivers and Vuong test (Rivers and Vuong, 2002). The null hypothesis H_0 is that two non-nested models are asymptotically equivalent, i.e.,

$$H_0: \lim_{n\to\infty} \left\{ \bar{Q}_n^1\left(\bar{\gamma}_n^1\right) - \bar{Q}_n^2\left(\bar{\gamma}_n^2\right) \right\} = 0, \tag{41}$$

[1] In fact we consider five models because we additionally use the DCC model of Tse and Tsui (2002) but its results are not significantly different from results of the DCC model of Engle that is why we do not present them.

Table 1
The results of parameters estimation for the four DCC models for selected currency rates.

Parameters	DCC-GARCH		DCC-CARR		DCC-RGARCH		DCC-OHLC	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
γ_{10}	−0.007	0.013	–	–	−0.008	0.013	−0.008	0.013
α_{10}	0.003	0.002	0.008	0.004	0.000	0.002	0.000	0.002
α_{11}	0.038	0.014	0.063	0.013	0.051	0.019	0.051	0.019
β_{11}	0.955	0.016	0.928	0.015	0.947	0.018	0.947	0.018
γ_{20}	0.000	0.026	–	–	−0.018	0.026	−0.018	0.026
α_{20}	0.008	0.005	0.015	0.007	0.002	0.006	0.002	0.006
α_{21}	0.064	0.013	0.120	0.016	0.091	0.020	0.091	0.020
β_{21}	0.936	0.013	0.874	0.017	0.908	0.018	0.908	0.018
γ_{30}	0.032	0.017	–	–	0.015	0.017	0.015	0.017
α_{30}	0.020	0.014	0.060	0.018	0.023	0.014	0.023	0.014
α_{31}	0.075	0.029	0.171	0.022	0.143	0.043	0.143	0.043
β_{31}	0.892	0.051	0.777	0.033	0.817	0.059	0.817	0.059
γ_{40}	−0.006	0.003	–	–	−0.007	0.003	−0.007	0.003
α_{40}	0.000	0.000	0.005	0.002	0.000	0.000	0.000	0.000
α_{41}	0.036	0.016	0.087	0.014	0.048	0.022	0.048	0.022
β_{41}	0.945	0.023	0.888	0.020	0.940	0.028	0.940	0.028
γ_{50}	0.039	0.013	–	–	0.010	0.012	0.010	0.012
α_{50}	0.020	0.008	0.0700	0.0153	0.018	0.012	0.018	0.012
α_{51}	0.155	0.035	0.3098	0.0320	0.325	0.092	0.325	0.092
β_{51}	0.805	0.043	0.6120	0.0439	0.659	0.104	0.659	0.104
ζ_1	0.010	0.002	0.008	0.002	0.009	0.002	0.015	0.010
θ_1	0.982	0.006	0.981	0.005	0.984	0.004	0.964	0.032

The sample period is from January 3, 2012, to December 31, 2018 (1004 observations). The γ_{k0} parameters are constants, α_{k0} α_{k1} β_{k1} are the parameters of the univariate GARCH model, the CARR model and the RGARCH model, $k = 1, 2, 3, 4, 5$ for EUR/USD, USD/JPY, GBP/USD, AUD/USD and USD/CAD, ζ_1, θ_1 are the parameters of the correlation part.

Table 2
The results of parameters estimation for the four DCC models for selected exchange-traded funds.

Parameters	DCC-GARCH		DCC-CARR		DCC-RGARCH		DCC-OHLC	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
γ_{10}	0.018	0.011	–	–	0.021	0.012	0.021	0.012
α_{10}	0.003	0.001	0.016	0.008	0.002	0.003	0.002	0.003
α_{11}	0.050	0.011	0.128	0.021	0.085	0.032	0.085	0.032
β_{11}	0.943	0.011	0.852	0.028	0.906	0.037	0.906	0.037
γ_{20}	0.019	0.010	–	–	0.020	0.010	0.020	0.010
α_{20}	0.002	0.001	0.008	0.003	0.002	0.001	0.002	0.001
α_{21}	0.031	0.007	0.080	0.010	0.047	0.011	0.047	0.011
β_{21}	0.961	0.008	0.909	0.012	0.940	0.015	0.940	0.015
γ_{30}	−0.008	0.013	–	–	−0.009	0.013	−0.009	0.013
α_{30}	0.003	0.002	0.0099	0.0041	0.005	0.002	0.005	0.002
α_{31}	0.030	0.006	0.0895	0.0110	0.050	0.013	0.050	0.013
β_{31}	0.961	0.007	0.9001	0.0129	0.931	0.017	0.931	0.017
γ_{40}	0.003	0.011	–	–	−0.002	0.011	−0.002	0.011
α_{40}	0.001	0.001	0.009	0.004	0.001	0.001	0.001	0.001
α_{41}	0.030	0.007	0.098	0.014	0.049	0.018	0.049	0.018
β_{41}	0.966	0.009	0.891	0.016	0.943	0.021	0.943	0.021
γ_{50}	0.003	0.011	–	–	−0.004	0.011	−0.004	0.011
α_{50}	0.001	0.001	0.006	0.003	0.001	0.002	0.001	0.002
α_{51}	0.027	0.006	0.076	0.011	0.037	0.012	0.037	0.012
β_{51}	0.968	0.007	0.916	0.013	0.955	0.015	0.955	0.015
ζ_1	0.024	0.005	0.024	0.005	0.025	0.005	0.030	0.009
θ_1	0.953	0.012	0.951	0.013	0.950	0.013	0.946	0.021

The sample period is from January 3, 2012, to December 31, 2018 (1037 observations). The γ_{k0} parameters are constants, α_{k0}, α_{k1}, β_{k1} are the parameters of the univariate GARCH model, the CARR model and the RGARCH model, $k = 1, 2, 3, 4, 5$ for SPDR Portfolio S&P 500 Growth, iShares Core U.S. Aggregate Bond, iShares U.S. Real Estate, United States Oil Fund, SPDR Gold Shares, respectively, ζ_1, θ_1 are the parameters of the correlation part.

where $Q_n^1\left(\gamma_n^1\right)$ and $Q_n^2\left(\gamma_n^2\right)$ is a selection criterion for two competing models, $\bar{Q}_n(\gamma_n)$ is expectation of $Q_n\left(\gamma_n\right)$, γ_n^1, γ_n^2 are parameter vectors associated with the considered models, $\bar{\gamma}_n^1$, $\bar{\gamma}_n^2$ are the so-called pseudo-true values and n is the sample size (see Vuong, 1989 for more details).

Table 3
The in-sample evaluation of the models and estimates of the conditional covariance matrix.

Evaluation criterion	DCC-GARCH	DCC-CARR	DCC-RGARCH	DCC-OHLC
Exchange-traded funds				
Log L	−5283.410	−5399.519	−5252.644	−5278.781
AIC	5.859	5.982	5.825	5.854
BIC	5.835	5.963	5.801	5.830
RV	–	−4.899	2.730	1.570
RV p-value	–	1.000	0.003	0.058
LF	0.559	0.571	0.540	0.523
SPA p-value	0.000	0.000	0.000	0.519
MCS p-value	0.000	0.000	0.000	1.000*
Currency rates				
Log L	−6229.325	−6518.036	−6176.399	−6194.951
AIC	7.112	7.435	7.052	7.073
BIC	7.087	7.415	7.027	7.048
RV	–	−3.358	2.401	0.267
RV p-value	–	1.000	0.008	0.395
LF	3.706	3.872	3.284	3.254
SPA p-value	0.003	0.000	0.001	0.594
MCS p-value	0.001	0.000	0.001	1.000*

The sample period is from January 3, 2012, to December 31, 2018 (1004 and 1037 observations for ETFs and currency rates, respectively). Log L is the logarithm of the likelihood function, RV is the Rivers-Vuong test statistic for model selection and comparisons are against the DCC-GARCH model, LF is the squared Frobenius loss function for the conditional covariance matrix.
*Indicates that models belong to the model confidence set with a confidence level of 0.90, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005).

The Rivers and Vuong test is a generalization of the Vuong tests (Vuong, 1989), which can be used for nonlinear models of time series. The values of the likelihood function and the results of the Rivers and Vuong test are given in Table 3. Additionally we present also the Akaike information criterion (AIC) and the Bayesian information criterion (BIC).

The highest values of the likelihood function and the lowest of the AIC and BIC criteria are for the DCC-RGARCH model. According to the Rivers and Vuong test, this model is asymptotically better than the benchmark DCC-GARCH model for both ETFs and currency pairs. Moreover, the DCC-OHLC model is asymptotically superior to the DCC-GARCH model at the 10% significance level for ETFs. However from the modeling and forecasting point of view, which model better describes the conditional covariance matrix is more important. Therefore, following Laurent et al. (2013), we evaluate the models based on the squared Frobenius loss function for the conditional covariance matrix. It can be formulated as:

$$\mathrm{LF} = (1/n)\sum_{t=1}^{n} \mathrm{Tr}\left[(\widehat{\mathbf{cov}}_t - \mathbf{rcov}_t)'(\widehat{\mathbf{cov}}_t - \mathbf{rcov}_t)\right], \tag{42}$$

where Tr is a trace of a matrix, $\widehat{\mathbf{cov}}_t$ is the estimated conditional covariance matrix, $\mathbf{rcov}_t$ is the realized covariance matrix calculated from intraday data (it is used as a proxy of the real conditional covariance matrix).

Both realized variances and covariances that are in the realized covariance matrix are calculated based on 5-min returns, excluding opening jumps. We also consider 15-min returns instead of 5-min returns and the conclusions are very similar to those presented in this paper. In order to evaluate whether the differences between the LF loss function for the competing models are statistically significant, we apply two tests: the test of superior predictive ability (SPA) of Hansen (2005) and the model confidence set (MCS) test of Hansen et al. (2011). In the first test, it is checked whether each of the models considered is outperformed significantly by any of the alternatives. In this regard, the performance of the benchmark model relative to model k can be described as:

$$d_{k,t} = \mathrm{LF}_{B,t} - \mathrm{LF}_{k,t},\ k = 1,\dots,m,\ t = 1,\dots,n, \tag{43}$$

where $\mathrm{LF}_{B,t}$ and $\mathrm{LF}_{k,t}$ are the squared Frobenius loss function for the conditional covariance matrix (formula (42)) from the benchmark model and model k, respectively, and m is the number of competing models (excluding the benchmark model). The null hypothesis of the SPA test is formulated as:

$$H_0 : E\left[d_{k,t}\right] \le 0,\ \text{for all } k = 1,\dots,m, \tag{44}$$

meaning that the benchmark model is not inferior to any of the models $k = 1,\dots,m$. The test statistic can be expressed as:

$$\mathrm{SPA} = \max_k \frac{\sqrt{T}\bar{d}_k}{\omega_k}, \tag{45}$$

where $\bar{d}_k$ is the mean of $d_{k,t}$ and ω_k^2 is a consistent estimator of the asymptotic variance.

The objective of the MCS procedure is to determine the set of best models, denoted as M_{best}, from a given collection of models, M. The set of the best models is defined as:

$$M_{best} \equiv \left\{ i \in M : E\left[d_{ij}\right] \le 0 \right\} \quad \text{for all } j \in M, \tag{46}$$

where $d_{ij} = \mathrm{LF}_{i,t} - \mathrm{LF}_{j,t}$ is the loss differential for $i, j \in M$.

Table 4
Evaluation of covariance matrix forecasts.

Model	Forecast evaluation criteria		
	LF	SPA p-value	MCS p-value
Exchange-traded funds			
DCC-GARCH	4.789	0.000	0.000
DCC-CARR	4.862	0.000	0.000
DCC-RGARCH	3.879	0.000	0.000
DCC-OHLC	2.932	0.573	1.000*
Currency rates			
DCC-GARCH	0.535	0.000	0.000
DCC-CARR	0.506	0.000	0.000
DCC-RGARCH	0.506	0.000	0.000
DCC-OHLC	0.444	0.570	1.000*

The evaluation period is January 4, 2016, to December 31, 2018 (754 and 774 forecasts for ETFs and currency rates, respectively). The realized variance is used as a proxy of variance and estimated as the sum of squares of 5-min returns, the realized covariance is used as a proxy of covariance and estimated as the sum of products of 5-min returns, LF is the squared Frobenius loss function for the conditional covariance matrix. We apply the SPA test four times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the first column of the table.
*Indicates that models belong to the model confidence set with a confidence level of 0.90, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005).

The null hypothesis is as follows:

$$H_0 : E\left[d_{ij,t}\right] = 0, \text{ for all } i, j \in M_s, \tag{47}$$

where $M_s \subset M$. The testing procedure begins with initially setting $M_s = M$. Then the null hypothesis is tested at a given significance level. If the null is not rejected then the $\widehat{M}_{best} = M_s$, otherwise the model that contributes most to the test statistic is removed from M_s and the whole procedure is repeated until there are no more models to be removed. The $\widehat{M}_{best}$ is then referred to as the model confidence set (MCS). The best models are selected with a given level of confidence in terms of a criterion for the loss function that is user-specified. In our case, we use the squared Frobenius loss function for the conditional covariance matrix (formula (42)).

The p-values for both tests are given in Table 3. The results of the SPA test indicate that the only model, which is not outperformed significantly by any of the alternatives is the DCC-OHLC model. According to the results of the MCS test, only the DCC-OHLC model belongs to the model confidence set. It clearly indicates that the most accurate estimates of the conditional covariance matrix are based on the proposed DCC model.

3.2. Out-of-sample forecasts

In this section, we compare the forecasting performance of the four analyzed multivariate GARCH models: DCC-GARCH, DCC-CARR, DCC-RGARCH and DCC-OHLC. For the starting sample (i.e., January 3, 2012 to December 31, 2015) we estimate the parameters of the models and compute one-day-ahead forecasts of the conditional covariance matrix. Consecutively, we add one new observation to the estimation sample while at the same time dropping the oldest observation. Then, based on the new estimation sample we re-estimate models and compute forecasts. The procedure is repeated until we obtain forecasts for the three years from January 4, 2016 to December 31, 2018.

The forecasts from the competing models are evaluated based on the squared Frobenius loss function defined in Eq. (42), however, the estimated conditional covariance matrix is replaced by its forecast. We check whether the differences in the forecasting performance among the DCC models are statistically significant by performing the SPA and MCS tests. The results of these tests are given in Table 4. We apply the SPA test four times, each time changing the model which is the benchmark. It means that the presented SPA p-value refers to the benchmark model specified in the first column of Table 4.

The lowest values of the squared Frobenius loss function, both for ETFs and currency rates, are for the DCC-OHLC model. The results of the SPA test indicate that the only model, which is not outperformed significantly by any of the alternatives is the DCC-OHLC model. Similar conclusions come from the MCS test. Only our proposed model belongs to the model confidence set. It means that the covariance matrix forecasts from the DCC-OHLC model are significantly more accurate than the forecasts based on the DCC-GARCH, DCC-CARR, DCC-RGARCH models for both ETFs and currency rates.

Additionally, we evaluate variance and covariance forecasts separately. The best forecasts are formulated based on the RGARCH and DCC-OHLC models for variance and covariance, respectively (these results are available from the authors upon request).

3.3. Value-at-risk forecasts

In this section, we check which of the considered models makes the most accurate forecasts of value-at-risk (VaR). VaR was developed by financial practitioners as an easily interpretable number that encodes information about a portfolio's risk. We formulate daily forecasts of VaR for two separate portfolios of exchange-traded funds and currency rates. All the portfolios are constructed with equal weights. The same assets and forecasting period are assumed as in the analysis of covariance matrices in Section 3.2

Following Abad et al. (2014) our evaluation is based on two approaches: the first involves testing the VaR forecasts for statistical accuracy, while the second relies on the measurement of the loss function to the economic agent. We test the statistical accuracy of the forecasts based on: the unconditional coverage test of Kupiec (1995), the independence and conditional coverage tests of Christoffersen (1998), and the unconditional coverage, independence and conditional coverage tests of Candelon et al. (2011). The results of the tests for the 95% VaR forecasts are given in Table 5 (the outcomes for the 99% confidence level are very similar and available from the authors upon request). The results for the Candelon et al. (2011) tests are presented for 5 moments, but we also obtained very similar results for 1, 2, 3, 4 and 6 moments.

None of the models provide fully satisfactory results for ETFs. On the other hand, all models pass the statistical criteria for currency rates. These statistical test results do not differ sufficiently among the competing models to clearly indicate which one is better. For that reason, we evaluate the models based on loss functions. Lopez (1998) suggested measuring the accuracy of VaR forecasts by the distance between observed returns and forecasted VaR. A model is penalized if a violation takes place and it is preferred to another one when it gives a lower loss value. In the general form he proposed the following formula:

$$LF_T = \begin{cases} f\left(r_T, VaR_T\right) \ if \ r_T \le VaR_T, \\ g\left(r_T, VaR_T\right) \ if \ r_T \ge -VaR_T, \end{cases} \tag{48}$$

where T is the forecast period, $f(x,y)$ and $g(x,y)$ are such functions that $f(x,y) \ge g(x,y)$. Summing LF_t over the back testing period we obtain

$$LF = \sum_{T=1}^{Nf} LF_T, \tag{49}$$

where Nf is the number of forecasts.

The best model is the one that minimizes (49). The regulator's loss function (RLF) takes into consideration the magnitude of the losses only when they occur, it is useful to evaluate the bank's internal models. We concentrate on regulator loss functions as the Basel Committee on Banking Supervision noted that the magnitude, as well as the number of VaR violations is a matter of regulatory concern (Basel Committee on Banking Supervision, 2011, 2019). We apply the loss function of Sarma et al. (2003) and three functions of Caporin (2008) given, respectively, as:

$$RLF(STS) = \begin{cases} \left(r_T - VaR_T\right)^2 \text{ if } r_T \le VaR_T, \\ 0 \text{ if } r_T \ge -VaR_T, \end{cases} \tag{50}$$

$$RLF(C1) = \begin{cases} \left|1 - \left|\dfrac{r_T}{VaR_T}\right|\right| \text{ if } r_T \le VaR_T, \\ 0 \text{ if } r_T \ge -VaR_T, \end{cases} \tag{51}$$

$$RLF(C2) = \begin{cases} \dfrac{\left(|r_T| - |VaR_T|\right)^2}{VaR_T} \text{ if } r_T \le VaR_T, \\ 0 \text{ if } r_T \ge -VaR_T, \end{cases} \tag{52}$$

$$RLF(C3) = \begin{cases} \left|r_T - VaR_T\right| \text{ if } r_T \le VaR_T, \\ 0 \text{ if } r_T \ge -VaR_T. \end{cases} \tag{53}$$

To assess whether the differences between values of loss functions are statistically significant, we perform the SPA and MCS tests. The results for the 95% VaR forecasts are given in Table 6 (the outcomes for the 99% confidence level are very similar and available from the authors upon request). For all loss functions we apply the SPA test four times, each time changing the model which is the benchmark. It means that the presented SPA p-value refers to the benchmark model specified in the heading of Table 6.

For all considered loss functions, significantly more accurate VaR forecasts are constructed based on the DCC-OHLC model than the DCC-GARCH, DCC-CARR and DCC-RGARCH models. The results are very similar for both commonly employed confidence levels, 95% and 99%. This means that the application of the proposed model can be beneficial for calculating risk measures.

4. Conclusions

Volatility models are largely based solely on closing prices, meanwhile, daily low and high prices significantly increase the amount of information about the variability of returns during a day. Low and high prices are almost always available with daily closing prices for financial series which is why their usage in volatility models is important from a practical point of view. In this study, we suggest a new specification of the DCC model based on OHLC prices (denoted by DCC-OHLC), which is a combination of the DCC model of Tse and Tsui (2002), the Range-GARCH model of Molnár (2016) and the correlation estimator of Popov (2016)

Table 5
Evaluation of 95% VaR forecasts: unconditional coverage and independence tests.

Statistic	DCC-GARCH		DCC-CARR		DCC-RGARCH		DCC-OHLC	
	Value	P-value	Value	P-value	Value	P-value	Value	P-value
Exchange-traded funds								
LR_{UC}	0.752	0.386	0.499	0.480	0.296	0.586	0.208	0.648
LR_{IND}	4.289	0.038	7.182	0.007	5.107	0.024	8.143	0.004
LR_{CC}	5.041	0.080	7.681	0.021	5.403	0.067	8.351	0.015
J_{UC}	0.950	0.333	0.688	0.413	0.463	0.463	0.085	0.744
J_{IND}	11.641	0.010	8.429	0.022	3.601	0.142	6.852	0.034
J_{CC}	10.231	0.042	8.889	0.055	3.927	0.256	7.430	0.076
Currency rates								
LR_{UC}	0.289	0.591	0.002	0.961	0.141	0.707	0.002	0.961
LR_{IND}	0.040	0.841	0.001	0.981	0.016	0.899	0.633	0.426
LR_{CC}	0.329	0.848	0.003	0.998	0.157	0.924	0.636	0.728
J_{UC}	0.453	0.478	0.044	0.832	0.273	0.592	0.044	0.823
J_{IND}	2.157	0.311	0.501	0.821	0.452	0.837	0.460	0.834
J_{CC}	1.931	0.579	0.510	0.941	0.551	0.924	0.468	0.946

The evaluation period is January 4, 2016, to December 31, 2018 (754 and 774 forecasts for ETFs and currency rates, respectively). LR_{UC} is the statistic for the Kupiec (1995) unconditional coverage test, LR_{IND} is the statistic for the Christoffersen (1998) independence test, LR_{CC} is the statistic for the Christoffersen (1998) conditional coverage test, J_{UC} is the statistic for the Candelon et al. (2011) unconditional coverage test, J_{IND} is the statistic for the Candelon et al. (2011) independence test for up to five lags, J_{CC} is the statistic for the Candelon et al. (2011) conditional coverage test with the number of moments fixed to 5, p-values for J_{UC}, J_{IND}, J_{CC} were corrected by Dufour's (2006) Monte Carlo procedure.

Table 6
Evaluation of 95% VaR forecasts: regulator loss functions tests.

Loss function	DCC-GARCH			DCC-CARR			DCC-RGARCH			DCC-OHLC		
	Value × 100	SPA p-value	MCS p-value	Value × 100	SPA p-value	MCS p-value	Value × 100	SPA p-value	MCS p-value	Value × 100	SPA p-value	MCS p-value
Exchange-traded funds												
RLF(STS)	0.863	0.017	0.002	0.920	0.006	0.004	0.812	0.005	0.004	**0.467**	0.590	1.000*
RLF(C1)	2.347	0.001	0.000	2.441	0.000	0.000	2.176	0.000	0.000	**1.325**	0.538	1.000*
RLF(C2)	1.220	0.001	0.000	1.341	0.000	0.006	1.147	0.000	0.000	**0.532**	0.553	1.000*
RLF(C3)	1.678	0.001	0.000	1.685	0.000	0.000	1.545	0.000	0.000	**1.115**	0.533	1.000*
Currency rates												
RLF(STS)	0.073	0.005	0.007	0.059	0.042	0.059	0.073	0.003	0.005	**0.053**	0.958	1.000*
RLF(C1)	1.517	0.000	0.000	1.285	0.009	0.009	1.477	0.000	0.000	**1.133**	0.511	1.000*
RLF(C2)	0.228	0.006	0.010	0.180	0.040	0.047	0.219	0.002	0.008	**0.153**	0.960	1.000*
RLF(C3)	0.487	0.000	0.000	0.421	0.012	0.015	0.474	0.000	0.000	**0.381**	0.507	1.000*

The evaluation period is January 4, 2016, to December 31, 2018 (754 and 774 forecasts for ETFs and currency rates, respectively). RLF(STS) is the loss function by Sarma et al. (2003), RLF(C1), RLF(C2), RLF(C3) are three loss functions by Caporin (2008). The lowest values of loss functions are marked in bold. For all loss functions we apply the SPA test four times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table.

*Indicates that models belong to the model confidence set with a confidence level of 0.90, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005).

based on OHLC prices. To the best of our knowledge, the proposed model is the first multivariate volatility model with the correlation estimator based on OHLC prices that can be applied to any assets for which such daily prices are available.

We compare the new model with the DCC-GARCH model of Engle (2002), the DCC-CARR model of Chou et al. (2009) and the DCC-RGARCH model of Fiszeder et al. (2019). We evaluate these models on two data sets: exchange-traded funds and currency rates.

The proposed model improves conditional covariance matrix estimates and increases the accuracy of the covariance matrix compared with the standard DCC model and two competing range-based DCC models, i.e., DCC-CARR and DCC-RGARCH. Moreover, VaR forecasts based on the DCC-OHLC model have no advantage over the forecasts based on other DCC models in terms of statistical accuracy, but they are more accurate under the loss functions to the economic agent. The advantage of the suggested model comes from the usage of the correlation estimator based on OHLC prices. Moreover, the main conclusions of the study are also robust to the forecast evaluation criterion employed.

In the future, the suggested approach can be extended to other multivariate GARCH or stochastic volatility models. For example, the newly proposed models such as the IDR-DCC-NL model of De Nard et al. (2021), the dynamic conditional angular correlation model of Jarjour and Chan (2020), the multivariate GARCH model with the dynamic beta of Raddant and Wagner (2021) would probably also benefit from applying the correlation estimator based on OHLC prices.

CRediT authorship contribution statement

Piotr Fiszeder: Conceptualization, Methodology, Data curation, Formal analysis, Writing – original draft, Writing – review & editing. **Marcin Fałdziński:** Conceptualization, Methodology, Data curation, Formal analysis, Writing – original draft, Writing – review & editing. **Peter Molnár:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing.

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