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Improving forecasts with the co-range dynamic conditional correlation model

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ABSTRACT

We introduce a new specification of the dynamic conditional correlation (DCC) model, where its parameters are estimated with the use of closing and additionally low and high prices. Such prices are often commonly available for many financial series and contain more information about the variation of returns. We construct the model with the range-based estimator of variance but more importantly also with the range-based estimator of covariance. The latter estimator and as a consequence the proposed DCC model require, however, that the range of a portfolio return is given. We compare the model with three other specifications of the DCC models and evaluate them based on Monte Carlo experiments and currencies rates from the Forex market. We show that the use of low and high prices can improve estimation of covariance and correlation matrices of returns and increase the accuracy of forecasts of covariance and correlation matrices based on this model, compared with using closing prices only. The proposed model is superior not only to the standard DCC model, but also to the competing range-based DCC model.

1. Introduction

Modelling relations between financial time series is of great practical importance. A multivariate framework leads to more relevant models than analysing separate univariate time series. The construction and valuation of portfolios of financial instruments and the management of their risk require the application of multivariate models. One of the most popular models to describe financial time series is the dynamic conditional correlation (DCC) model introduced independently by Engle (2002) and Tse and Tsui (2002). The advantages of these formulations are the positive definiteness of the conditional covariance matrix and the ability to describe time-varying conditional correlations between returns of assets. Parameters of the DCC model can be estimated in two stages, which makes this approach relatively simple and possible to apply even for very large portfolios.

The analysis of multivariate volatility models is largely based solely on closing prices. Meanwhile, the application of low and high prices in univariate volatility models improves volatility estimation and increases the accuracy of volatility forecasts compared with models based only on closing prices (see e.g. Alizadeh et al., 2002; Chou, 2005; Brandt and Jones, 2006; Li and Hong, 2011; Fiszeder and Perczak, 2016; Molnár, 2016). The extension of univariate studies to multivariate ones is

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therefore much desired. The idea of the construction of multivariate volatility models based on low and high prices consists in applying univariate model specifications based on the price range and incorporating them into multivariate models of the covariance matrix of closing returns (see Chou and Cai, 2009, Chou et al., 2009; Asai, 2013; Su and Wu, 2014 for the GARCH models or Harris and Yilmaz, 2010 for the EWMA model). The issue is important from a practical viewpoint, because daily low and high prices are often commonly available with closing prices for financial series. The application of such prices has also economic consequences (see Chou and Liu, 2010; Wu and Liang, 2011).

We carry out Monte Carlo experiments by fitting the four DCC models formulated on closing or low and high prices to systems that are simulated by a very general multivariate GARCH model and analyze the implications of the usage of different kind of data for the estimation and prediction of conditional covariance and correlation matrices. Additionally we perform an empirical analysis for the most heavily traded currency pairs in the Forex market.

This study has three main contributions. The first one is a proposition of a new specification of the DCC model (denoted by co-range DCC), where its parameters are estimated with the use of closing and additionally low and high prices. Low and high prices have been already used in the DCC models by Chou et al. (2009), Asai (2013), Su and Wu (2014) but in this paper, a different approach is suggested. We use the range-based univariate volatility model CARR (the conditional autoregressive range model introduced by Chou, 2005) instead of the GARCH model in the first stage of estimation do describe the volatility. However, at the same time the range-based estimator of the covariance of returns is applied in the DCC model of Tse and Tsui (2002) in the second stage of estimation. The range-based estimator of the covariance of returns has been already applied in the multivariate CARR model proposed by Fernandes et al. (2005) but their specification is not parsimonious and estimation of its parameters is difficult. That is why this model is not used in empirical research. Even the authors of the model have not applied it in any empirical study. Our parameterization is based on the DCC model that is why estimation of its parameters is relatively straightforward. The range-based estimator of covariance and as a consequence the proposed DCC model require that the range of a portfolio return is given. This range can be calculated in some particular cases, for example, when cross rates of foreign exchange rates are given or when tick-by-tick data are available (the range-based estimator can be less sensitive than the realized covariance to some sources of the microstructure noise arising from the bid-ask spread and nonsynchronous trading; see Brandt and Diebold, 2006).

The second contribution is to demonstrate that the use of additional information about low and high prices in the formulation of the DCC model can improve estimation of covariance and correlation matrices of returns and increase the accuracy of forecasts of covariance and correlation matrices based on this model, compared with using closing prices only.

Third, we show that forecasts of covariance and correlation matrices based on the proposed model are more accurate than the ones from the range-based DCC model of Chou et al. (2009), which is also formulated with the use of low and high prices and is the main competitor of the co-range DCC model in this class of models. Because both models apply the same CARR model in the first stage of estimation, it means that the superiority of the proposed model in covariance forecasting results from the application of the range-based estimator of covariance in the second stage of estimation.

The plan for the rest of the paper is as follows. In Section 2 a description of applied models is provided. Section 3 compares them by carrying out Monte Carlo experiments to analyse the effects of their specifications on the estimation and forecasting of conditional covariance and correlation matrices. An empirical application to the exchange rate returns of the euro (EUR), Japanese yen (JPY) and British pound (GBP) against the U.S. dollar (USD) is carried out in Section 4. Summary is given in Section 5.

2. Methods applied

Due to many advantages, presented in the Introduction, the DCC model is one of the most popular multivariate volatility model. As it was said the model was introduced independently by Engle (2002) and Tse and Tsui (2002). Both models are based on returns calculated from closing prices. The other two analysed models, i.e., the range-based DCC (Chou et al., 2009) and the proposed co-range DCC model are formulated with the usage of low and high prices.

2.1. The DCC model of Engle (2002)

Let us assume that the $\boldsymbol{\epsilon}_t$ ($N \times 1$ vector) is the multivariate innovation process for the conditional mean (or in a particular case the multivariate return process) and can be written as:

$$\boldsymbol{\varepsilon}_t|\psi_{t-1} \sim Normal(0,\ \mathbf{cov}_t), \tag{1}$$

where ψ_{t-1} is the set of all information available at time $t-1$, *Normal* is the conditional multivariate normal distribution and $\mathbf{cov}_t$ is the $N \times N$ symmetric conditional covariance matrix.

The DCC(1,1) model of Engle (2002) can be presented as (for the sake of simplicity, all models in the paper are presented with lag one of past squared returns, conditional variances and covariances):

$$\mathbf{Q}_t = (1-\theta_1-\theta_2)\mathbf{S} + \theta_1\left(\mathbf{z}_{t-1}\mathbf{z}'_{t-1}\right) + \theta_2\mathbf{Q}_{t-1}, \tag{2}$$

$$\mathbf{cor}_t = \mathbf{Q}_t^{*-1}\mathbf{Q}_t\mathbf{Q}_t^{*-1}, \tag{3}$$

$$\mathbf{cov}_t = \mathbf{D}_t\mathbf{cor}_t\mathbf{D}_t, \tag{4}$$

where $\mathbf{z}_t$ is the standardised $N \times 1$ residual vector assumed to be serially independently distributed given as $\mathbf{z}_t = \mathbf{D}_t^{-1}\boldsymbol{\varepsilon}_t$, $\mathbf{D}_t = \mathrm{diag}(h_{1t}^{1/2}, h_{2t}^{1/2}, \ldots, h_{Nt}^{1/2})$, conditional variances h_{kt} (for $k = 1, 2, \ldots, N$) are described as univariate GARCH models Eqs. (5)–(6), according to Engle (2002) $\mathbf{S}$ is the unconditional $N \times N$ correlation matrix of $\mathbf{z}_t$, $\mathbf{cor}_t$ is the time varying $N \times N$ conditional correlation matrix of $\mathbf{z}_t$, $\mathbf{Q}_t^*$ is the diagonal $N \times N$ matrix composed of the square root of the diagonal elements of $\mathbf{Q}_t$.

A sufficient condition for the positivity of $\mathbf{cov}_t$ is that all conditional variances are positive, the parameters $\theta_1, \theta_2 > 0$ and satisfy the condition $\theta_1 + \theta_2 < 1$. Aielli (2013) points out that $\mathbf{S}$ is not the correlation matrix of $\mathbf{z}_t$ and proposes the corrected DCC model (cDCC). The performance of the cDCC and DCC models does not, however, differ much in practice for typical financial series (see e.g. Bauwens et al., 2013, Bauwens and Otranto, 2016; de Almeida et al., 2018), that is why we use the standard model of Engle (2002) as our benchmark.

The univariate GARCH(1,1) model (introduced by Bollerslev, 1986) used in the DCC model can be written as:

$$\varepsilon_{kt}|\psi_{t-1} \sim Normal(0,\ h_{kt}), \quad k = 1, 2, \ldots, N, \tag{5}$$

$$h_{kt} = \alpha_{k0} + \alpha_{k1}\varepsilon_{k\ t-1}^2 + \beta_{k1}h_{k\ t-1}, \tag{6}$$

where $\alpha_{k0} > 0, \alpha_{k1} \geq 0, \beta_{k1} \geq 0$ (for $k = 1, 2, \ldots, N$). The requirement for covariance stationarity of ε_{kt} is $\alpha_{k1} + \beta_{k1} < 1$.

A nice feature of the DCC-GARCH model is that its parameters can be estimated by the quasi-maximum likelihood method using a two-stage approach. Let the parameters of the model $\boldsymbol{\Theta}$ be written in two groups $\boldsymbol{\Theta}' = (\boldsymbol{\Theta}'_1, \boldsymbol{\Theta}'_2)$, where $\boldsymbol{\Theta}_1$ is the vector of parameters of conditional means and variances and $\boldsymbol{\Theta}_2$ is the vector of parameters of the correlation part of the model. The log-likelihood function can be written as the sum of two parts:

$$L(\boldsymbol{\Theta}) = L_{Vol}(\boldsymbol{\Theta}_1) + L_{Corr}(\boldsymbol{\Theta}_2|\boldsymbol{\Theta}_1), \tag{7}$$

where $L_{Vol}(\boldsymbol{\Theta}_1)$ represents the volatility part:

$$L_{Vol}(\boldsymbol{\Theta}_1) = -\frac{1}{2}\sum_{t=1}^{n}\left(N\ln(2\pi) + \ln|\mathbf{D}_t|^2 + \boldsymbol{\varepsilon}_t'\mathbf{D}_t^{-2}\boldsymbol{\varepsilon}_t\right), \tag{8}$$

while $L_{Corr}(\boldsymbol{\Theta}_2|\boldsymbol{\Theta}_1)$ can be viewed as the correlation component:

$$L_{Corr}(\boldsymbol{\Theta}_2|\boldsymbol{\Theta}_1) = -\frac{1}{2}\sum_{t=1}^{n}\left(\ln|\mathbf{cor}_t| + \mathbf{z}_t'\mathbf{cor}_t^{-1}\mathbf{z}_t - \mathbf{z}_t'\mathbf{z}_t\right). \tag{9}$$

$L_{Vol}(\boldsymbol{\Theta}_1)$ can be written as the sum of log-likelihood functions of N univariate GARCH models:

$$L_{Vol}(\boldsymbol{\Theta}_1) = -\frac{1}{2}\sum_{k=1}^{N}\left(n\ln(2\pi) + \sum_{t=1}^{n}\left(\ln(h_{kt}) + \frac{\epsilon_{kt}^2}{h_{kt}}\right)\right). \tag{10}$$

It means that in the first stage the parameters of univariate GARCH models can be estimated separately for each of the assets and the estimates of h_{kt} can be obtained. In the second stage residuals transformed by their estimated standard deviations are used to estimate the parameters of the correlation part ($\boldsymbol{\Theta}_2$) conditioning on the parameters estimated in the first stage ($\widehat{\Theta}_1$). In practice, the described procedure is a three-stage approach, because the unconditional correlation matrix $\mathbf{S}$ has to be estimated before.

The DCC model is one of the most popular multivariate GARCH models used to describe financial time series. Caporin and McAleer (2013) highlight some critical issues connected with the lack of statistical properties of this model. They are not, however, against the use of DCC model, but suggest to apply it as a filter in forecasting out-of-sample conditional covariances and correlations. Moreover, many papers like Laurent et al. (2012), Bauwens et al. (2013), Noureldin et al. (2014), de Almeida et al. (2018) show that it is very difficult to outperform the DCC model by other multivariate GARCH models.

2.2. The DCC model of Tse and Tsui (2002)

The main difference between DCC models of Engle (2002) and Tse and Tsui (2002) is a formulation of the correlation matrix. In the model of Tse and Tsui conditional correlations are the weighted sum of past conditional correlations, while in the model of Engle the matrix written like the GARCH equation is later transformed to the correlation matrix. In this paper the model of Tse and Tsui was extended because it was easier to estimate the parameters of such new model. Moreover, the DCC model of Engle, in contrast to the model of Tse and Tsui, provide estimated dynamic correlations as a product of standardization, and not as a direct result of the equation governing the multivariate dynamics (see Caporin and McAleer, 2013).

The DCC(1,1) model of Tse and Tsui (2002) can be presented as:

$$\boldsymbol{\varepsilon}_t|\psi_{t-1} \sim Normal(0,\ \mathbf{cov}_t), \tag{11}$$

$$\boldsymbol{\Xi}_{t-1} = \mathbf{B}_{t-1}^{-1}\mathbf{L}_{t-1}\mathbf{L}'_{t-1}\mathbf{B}_{t-1}^{-1}, \tag{12}$$

$$\mathbf{cor}_t = (1 - \theta_1 - \theta_2)\mathbf{cor} + \theta_1\boldsymbol{\Xi}_{t-1} + \theta_2\mathbf{cor}_{t-1}, \tag{13}$$

$$\mathbf{cov}_t = \mathbf{D}_t \mathbf{cor}_t \mathbf{D}_t, \tag{14}$$

where $\boldsymbol{\Xi}_{t-1}$ is the $N \times N$ sample estimate of the conditional correlation matrix based on recent M standardised residuals $\{\mathbf{z}_{t-1}, \mathbf{z}_{t-2}, \ldots, \mathbf{z}_{t-M}\}$, $\mathbf{z}_t$ is the standardised $N \times 1$ residual vector assumed to be serially independently distributed given as $\mathbf{z}_t = \mathbf{D}_t^{-1}\boldsymbol{\varepsilon}_t$, $\mathbf{D}_t = \mathrm{diag}(h_{1t}^{1/2}, h_{2t}^{1/2}, \ldots, h_{Nt}^{1/2})$, conditional variances h_{kt} (for $k = 1, 2, \ldots, N$) are described as univariate GARCH models, $\mathbf{B}_{t-1}$ is the $N \times N$ diagonal matrix with the kth diagonal element being $(\sum_{h=1}^{M} z_{k\ t-h}^2)^{0.5}$, $z_{kt} = \varepsilon_{kt}/\sqrt{h_{kt}}$, $\mathbf{L}_{t-1}$ is the $N \times M$ matrix given as $\mathbf{L}_{t-1} = (\mathbf{z}_{t-1}, \ldots, \mathbf{z}_{t-M})$, $\mathbf{cor}_t$ is the conditional $N \times N$ correlation matrix of ε_t, $\mathbf{cor}$ is the unconditional sample $N \times N$ correlation matrix, parameters θ_1, θ_2 are nonnegative and satisfy the condition $\theta_1 + \theta_2 < 1$.

Denoting $\boldsymbol{\Xi}_t = \{\xi_{ijt}\}$, the ijth element of $\boldsymbol{\Xi}_{t-1}$ is given as:

$$\xi_{ij\ t-1} = \frac{\sum_{h=1}^{M} z_{it-h} z_{jt-h}}{\sqrt{\left(\sum_{h=1}^{M} z_{it-h}^2\right)\left(\sum_{h=1}^{M} z_{jt-h}^2\right)}}. \tag{15}$$

The positive definiteness of $\mathbf{cor}_t$ is ensured by construction if $\mathbf{cor}_0$ and $\boldsymbol{\Xi}_{t-1}$ are positive definite. A necessary condition for the latter to hold is $M \geq N$.

The parameters of the DCC model of Tse and Tsui (2002) can be estimated by the quasi-maximum likelihood method using a two-stage approach. The log-likelihood function is the same as in the DCC model of Engle (2002). Tse and Tsui (2002) estimated the parameters of their model in one stage. Such estimators are more efficient than those obtained by a two-stage approach.

2.3. The CARR model

The two additional models applied in the paper, i.e., the range-based DCC and the co-range DCC use the CARR model of Chou (2005) instead of the GARCH model in the first stage of estimation. That is why this model is described in this subsection.

Let assume that H_t and L_t are high and low prices over a fixed period such as day, week or month and the observed price range is given as $R_t = \ln(H_t) - \ln(L_t)$. The CARR(1,1) model can be described as:

$$R_t = \lambda_t u_t, \tag{16}$$

$$u_t | \psi_{t-1} \sim \exp(1, \xi_t), \tag{17}$$

$$\lambda_t = \varphi_0 + \varphi_1 R_{t-1} + \varphi_2 \lambda_{t-1}, \tag{18}$$

where λ_t is the conditional mean of the range and u_t is the disturbance term.

The exponential distribution is a natural choice for the conditional distribution of u_t because it takes positive values. The parameters φ_0, φ_1, φ_2 should be positive to ensure the positivity of λ_t.

The process is covariance stationary if the following condition is met:

$$\varphi_1 + \varphi_2 < 1. \tag{19}$$

It is worth emphasizing that the CARR model describes the dynamics of the conditional mean of the price range, not the conditional variance of returns as in the case of the GARCH model.

The parameters of the CARR model can be estimated by the quasi-maximum likelihood method. The log-likelihood function can be written as:

$$L(\varsigma) = -\sum_{t=1}^{n} \left(\ln\lambda_t + \frac{R_t}{\lambda_t} \right), \tag{20}$$

where ς is a vector containing unknown parameters of the model.

The estimators obtained by the quasi-maximum likelihood method are consistent (see Engle and Russell, 1998, Engle, 2002 and Chou, 2005).

2.4. The range-based DCC model

Chou et al. (2009) combined the CARR model of Chou (2005) with the DCC model of Engle (2002) to propose the range-based DCC model. The CARR model describes the dynamics of the conditional mean of the price range, that is why in order to estimate the conditional standard deviation of returns the conditional price range has to be scaled according to the formula: $\lambda_{kt}^* = \mathrm{adj}_k \lambda_{kt}$ for $k = 1, 2, \ldots N$, where $\mathrm{adj}_k = \bar{\sigma}_k / \bar{\lambda}_k$. The scaling factor adj_k is estimated as the quotient of unconditional standard deviation of returns by the sample mean of the conditional range.

The range-based DCC(1,1) model can be expressed as:

$$\boldsymbol{\varepsilon}_t | \psi_{t-1} \sim Normal(0,\ \mathbf{cov}_t), \tag{21}$$

$$\mathbf{Q}_t = (1 - \theta_1 - \theta_2)\mathbf{S} + \theta_1 (\mathbf{z}_{t-1}^{CARR} (\mathbf{z}_{t-1}^{CARR})') + \theta_2 \mathbf{Q}_{t-1}, \tag{22}$$

$$\mathbf{cor}_t = \mathbf{Q}_t^{*-1}\mathbf{Q}_t\mathbf{Q}_t^{*-1}, \tag{23}$$

$$\mathbf{cov}_t = \mathbf{D}_t\mathbf{cor}_t\mathbf{D}_t, \tag{24}$$

where $\mathbf{D}_t = \text{diag}(\lambda_{1t}^*, \lambda_{2t}^*, ..., \lambda_{Nt}^*)$, $\mathbf{z}_t^{CARR}$ is the standardised $N \times 1$ residual vector, which contains the standardised residuals z_{kt}^{CARR} calculated from the CARR model Eqs. (16)–((18)) as $z_{kt}^{\text{CARR}} = \varepsilon_{kt}/\lambda_{kt}^*$, the other variables are defined in the same way as in the DCC-GARCH model. The parameters θ_1, θ_2 are nonnegative and satisfy the condition $\theta_1 + \theta_2 \leq 1$.

The parameters of the range-based DCC model can be estimated by the quasi-maximum likelihood method using a two-stage approach. The log-likelihood function can be written as the sum of two parts:

$$L^{range\ DCC}(\mathbf{\Theta}) = L_{Vol}^{range\ DCC}(\mathbf{\Theta}_1) + L_{Corr}^{range\ DCC}(\mathbf{\Theta}_2|\mathbf{\Theta}_1), \tag{25}$$

where the volatility part is expressed as:

$$L_{Vol}^{range\ DCC}(\mathbf{\Theta}_1) = -\frac{1}{2}\sum_{k=1}^{N}\left(n\ln(2\pi) + \sum_{t=1}^{n}\left(2\ln\left(\lambda_{kt}^*\right) + \frac{\epsilon_{kt}^2}{\lambda_{kt}^{*2}}\right)\right) \tag{26}$$

and the correlation component has the form:

$$L_{Corr}^{range\ DCC}(\mathbf{\Theta}_2|\mathbf{\Theta}_1) = -\frac{1}{2}\sum_{t=1}^{n}\left(\ln|\mathbf{cor}_t| + (\mathbf{z}_t^{CARR})'\mathbf{cor}_t^{-1}\mathbf{z}_t^{CARR} - (\mathbf{z}_t^{CARR})'\mathbf{z}_t^{CARR}\right). \tag{27}$$

This means that in the first stage the parameters of the CARR models can be estimated separately for each of the assets. In the second stage the standardised residuals z_{kt}^{CARR} are used to maximize Eq. (27) in order to estimate the parameters of the correlation component.

2.5. The estimator of covariance based on low and high prices

The range-based covariance estimator is applied in the proposed co-range DCC model (this estimator was also used in the proposition of the new specification of the BEKK model with high and low prices by Fiszeder, 2018). That is why it is described in more detail in this subsection. One can expect that such estimator has advantage over the one based on only closing prices (see Brunetti and Lildholdt, 2002). Let us assume that the two assets have a similar daily pattern (i.e., they move in the same direction throughout the day). It implies a high daily covariance between the assets, which can be captured by the estimator based on low and high prices. If the daily return calculated on the basis of closing prices is close to zero for both assets, than the estimate of covariance based on daily returns is also close to zero and will fail to capture the daily comovements.

The suggested approach, considered by Brunetti and Lildholdt (2002), Fernandes et al. (2005) and Brandt and Diebold (2006), is based on the transformed formula for the variance of the sum of two random variables. The range-based covariance estimator can be expressed as:

$$\text{cov}(X, Y) = [\text{var}(X+Y) - \text{var}(X) - \text{var}(Y)]/2, \tag{28}$$

where all variances are calculated based on range-based estimators. Different estimators based on daily low, high or additionally open and closing prices (like Parkinson, 1980; Garman and Klass, 1980 or Rogers and Satchell, 1991) can be applied in the above formula. For an overview of range-based volatility estimators see for example Molnár (2012), Fiszeder and Perczak (2013), Chou et al. (2014).

This method of covariance estimation is possible when the range of a portfolio return is given (the variance for the sum of variables X and Y is needed). Such data are unfortunately rarely available (see for some examples in Brandt and Diebold, 2006). The range of a portfolio return can be, however, calculated based on tick-by-tick data. The realized covariance, i.e., the estimator of covariance constructed based on intraday prices is more efficient than the estimator based on high and low prices (similarly like the realised variance is more efficient than the range-based estimators of variance), but the latter can be less sensitive to some sources of the microstructure noise arising from the bid-ask spread and non-synchronous trading (see Monte Carlo simulations performed by Brandt and Diebold, 2006). For these reasons, in case when the effects of the microstructure noise on the realized covariance are severe, the range-based estimator of covariance can be applied as one of more robust alternatives.

The range of a portfolio return can be calculated without the use of tick-by-tick data in the case of foreign exchange rates. Consider two exchange rates of currencies A and B in terms of currency C, denoted as A/C and B/C. In the absence of triangular arbitrage, the cross rate can be given as:

$$\text{A}/B = \frac{\text{A/C}}{\text{B/C}} \tag{29}$$

and written for returns has the form:

$$\Delta\text{lnA/B} = \Delta\text{lnA/C} - \Delta\text{lnB/C}. \tag{30}$$

The estimator of the covariance of returns is then expressed as:

$$\mathrm{cov}(\Delta \mathrm{lnA/C}, \Delta \mathrm{lnB/C}) = [\mathrm{var}(\Delta \mathrm{lnA/C}) + \mathrm{var}(\Delta \mathrm{lnB/C}) - \mathrm{var}(\Delta \mathrm{lnA/B})]/2. \tag{31}$$

The idea of using triangular arbitrage in order to calculate the covariance of returns is not new and has been already employed for example in Lopez and Walter (2001) for the implied covariance, in Brunetti and Lildholdt (2002) for the co-range, in Andersen et al. (2003) for the realized covariance, in Harris and Yilmaz (2010) for the imputed multivariate EWMA model.

Monte Carlo simulations performed by Brunetti and Lildholdt (2002) and Brandt and Diebold (2006) indicate that the estimator of the range-based covariance of returns is biased (downward for positive correlation and upward for negative correlation) but highly efficient (approximately five times more efficient than the estimator based on closing prices when the Parkinson estimator is applied). The downward bias is also present for correlations (Brandt and Diebold, 2006). The bias of the range-based estimators can be, however, eliminated in a similar fashion as it is corrected by Chou et al. (2009) for the price range or by Martens and van Dijk (2007) for the realized range.

2.6. The DCC model with the range-based covariance of returns

The new formulation of the DCC model with the usage of low and high prices is proposed here. The DCC models are constructed to permit for two-stage estimation of parameters. We use the CARR model described in Section 2.3 (Eqs. (16)–(18)) instead of the GARCH model in the first stage. As it was said the CARR model was used for the first time in the DCC model of Engle (2002) by Chou et al. (2009) in their range-based DCC model. We go one step further and apply the range-based covariance estimator (described in Section 2.5; Eq. (28)) and the range-based variance estimator in the second stage of estimation.

The DCC(1,1) model with low and high prices (denoted by co-range DCC) can be given as:

$$\boldsymbol{\varepsilon}_t|\psi_{t-1} \sim Normal(0,\ \mathbf{cov}_t), \tag{32}$$

$$\mathbf{cor}_t = (1-\theta_1-\theta_2)\mathbf{cor} + \theta_1\boldsymbol{\Phi}_{t-1} + \theta_2\mathbf{cor}_{t-1}, \tag{33}$$

$$\mathbf{cov}_t = \mathbf{D}_t\mathbf{cor}_t\mathbf{D}_t, \tag{34}$$

where $\boldsymbol{\Phi}_{t-1}$ is the $N\times N$ correlation matrix of returns calculated with the use of low and high prices with the ijth element given as $\phi_{ijt-1} = \mathrm{cov}_{ijt-1}/(\sigma_{it-1}\sigma_{jt-1})$, i.e., the range-based covariance of returns given by formula (28) is divided by the product of the range-based standard deviations of returns. Parameters θ_1, θ_2 are nonnegative and satisfy the condition $\theta_1+\theta_2 \le 1$.

In comparison to the DCC model of Tse and Tsui (2002), instead of the correlation matrix $\boldsymbol{\Xi}_{t-1}$ calculated on the basis of closing prices, we apply the matrix $\boldsymbol{\Phi}_{t-1}$ of more efficient estimators of correlations formulated with the use of low and high prices. Different range-based variance estimators based on daily low, high or additionally open and closing prices can be applied for this purpose. In the simulation study and empirical application in the subsequent sections the estimator of the covariance of returns in formula (31) and the Parkinson estimator (Parkinson, 1980) was used. The latter can be expressed as:

$$\sigma_{Pt}^2 = [\ln(H_t/L_t)]^2/(4\ln 2), \tag{35}$$

where H_t and L_t are daily high and low prices.

The positive definiteness of $\mathbf{cor}_t$ in formula (33) is ensured by construction if $\mathbf{cor}_0$ and $\boldsymbol{\Phi}_{t-1}$ are positive definite. It should be noted that the proposed model is parsimonious and there are no additional parameters relative to the model based only on returns of closing prices.

The parameters of the co-range DCC model can be estimated by the quasi-maximum likelihood method using a two-stage approach. The log-likelihood function can be written as the sum of two parts:

$$L^{co-range\ DCC}(\boldsymbol{\Theta}) = L_{Vol}^{co-range\ DCC}(\boldsymbol{\Theta}_1) + L_{Corr}^{co-range\ DCC}(\boldsymbol{\Theta}_2|\boldsymbol{\Theta}_1), \tag{36}$$

where the volatility part is expressed as:

$$L_{Vol}^{co-range\ DCC}(\boldsymbol{\Theta}_1) = -\frac{1}{2}\sum_{k=1}^{N}\left(n\ln(2\pi) + \sum_{t=1}^{n}\left(2\ln\left(\lambda_{kt}^{*}\right) + \frac{\epsilon_{kt}^2}{\lambda_{kt}^{*2}}\right)\right) \tag{37}$$

and the correlation component has the form:

$$L_{Corr}^{co-range\ DCC}(\boldsymbol{\Theta}_2|\boldsymbol{\Theta}_1) = -\frac{1}{2}\sum_{t=1}^{n}\left(\ln|\mathbf{cor}_t| + (\boldsymbol{z}_t^{CARR})'\mathbf{cor}_t^{-1}\boldsymbol{z}_t^{CARR} - (\boldsymbol{z}_t^{CARR})'\boldsymbol{z}_t^{CARR}\right), \tag{38}$$

the $\boldsymbol{z}_t^{CARR}$ is the $N\times 1$ residual vector, which contains the standardised residuals z_{kt}^{CARR} calculated from the CARR models.

It means that in the first stage the parameters of univariate CARR models can be estimated separately for each of the assets. In the second stage the standardised residuals z_{kt}^{CARR} are used to maximize Eq. (38) in order to estimate the parameters of the correlation component.

3. The Monte Carlo simulation

In this section we conducted Monte Carlo experiments in order to analyse the properties of the proposed co-range DCC model and its competitors. In particular, we investigated the properties of predictions of conditional variances and covariances that are obtained by fitting considered DCC models to the systems of daily conditional covariance matrices generated by the VECH model with intraday prices described by the geometric Brownian motion. The analysis of the forecasting performance on simulated data is especially important for volatility models, because the real values of covariances and variances are unobservable in practice and have to be estimated. It can cause serious errors in a forecasting evaluation (see e.g. Patton and Sheppard, 2009, Patton, 2011; Violante and Laurent, 2012).

Daily conditional covariance matrices were simulated by the VECH model. It is a very general multivariate GARCH model, proposed by Bollerslev et al. (1988), in which all of the variances and covariances are interrelated. The VECH model can be presented as:

$$\boldsymbol{\varepsilon}_t|\psi_{t-1} \sim Normal(0,\ \mathbf{cov}_t), \tag{39}$$

$$\text{vech}(\mathbf{cov}_t) = \mathbf{C} + \mathbf{A}\ \text{vech}\left(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}\right) + \mathbf{B}\ \text{vech}(\mathbf{cov}_{t-1}), \tag{40}$$

where $\mathbf{C}$ is the $N(N+1)/2\ \times\ 1$ vector of constants, $\mathbf{A}$ and $\mathbf{B}$ are the $N(N+1)/2\ \times N(N+1)/2$ parameter matrices, vech(·) denotes the operator that stacks the columns of the lower triangular part of the $N \times N$ matrix as the $N(N+1)/2\ \times\ 1$ vector.

The VECH process is covariance stationary if the moduli of the eigenvalues of $\mathbf{A}+\mathbf{B}$ are less than one (see Engle and Kroner, 1995). The unconditional covariance matrix $\boldsymbol{\Sigma} = E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}'_t)$ can be formulated than as:

$$\text{vech}(\boldsymbol{\Sigma}) = \left(\mathbf{I}_{N(N+1)/2} - \mathbf{A} - \mathbf{B}\right)^{-1}\mathbf{C}. \tag{41}$$

Gourieroux (1997), Francq and Zakoïan (2010), Chrétien and Ortega (2014) discuss sufficient conditions for the positivity of the conditional covariance matrix.

The VECH model was chosen as the data generating process because of its generality. It allows the simulated systems to have realistic complex dynamics that are similar to those usually observed when dealing with financial time series, which are often characterized by volatility spillovers and time-varying correlations. The chosen volatility model does not favour directly either of the competing DCC models.

We simulated conditional covariance matrices generated by the five-variate VECH model of a length of 501 daily observations ($N = 5,\ n = 501$) with the Student's t-distribution with seven degrees of freedom and multivariate Gaussian distribution in (39) (in fact at the beginning we simulated additional 5 observations in order to start estimation of the DCC model of Tse and Tsui (2002)). Data generated by the model with the Student's t-distribution have fatter tails and are more similar to real financial series, that is why, in the next subsections, only the results for the Student's t-distribution were presented, but similar results were also obtained for the Gaussian distribution. The lack of relations in conditional means of returns was assumed. We applied 1,000 repetitions in the Monte Carlo simulations. The values of the VECH parameters were similar to the ones in the paper of de Almeida et al. (2018) and are reported in Appendix. They have been chosen to satisfy the stationarity and positivity restrictions, while simultaneously representing the parameters that are often obtained when multivariate GARCH models are fitted to returns of exchange rates and stock indices. First 500 observations were used for the in-sample analysis, and the last, i.e., the 501st observation was used for the evaluation of the out-of-sample performance for one-day-ahead forecasts of covariance and correlation matrices.

In order to estimate the parameters of the proposed model low and high prices are also necessary. For that reason, for each day we simulated 5 series of 100,000 intraday prices, based on the geometric Brownian motion with zero drift. Such assumption means that intraday log-returns have normal distribution. The geometric Brownian motion was assumed as the data generating process for intraday prices in most papers on range-based estimators. Each day variances and covariances were determined by the conditional covariance matrix $\mathbf{cov}_t$ from the VECH model. For all pairs of series the cross rate was generated on the assumption of the absence of triangular arbitrage. The white noise process with variance equal to 6.3e-10 was added in Eq. (29). Its value was estimated based on exchange rates presented in Section 4 and it is connected with the liquidity of the market (we tried also different values but it did not influence the results significantly. In total, 10 cross rates time series were simulated. For all intraday series (15 rates) daily low, high and closing prices were recorded and these prices were used to estimate the parameters of the competing DCC models in the next subsections.

3.1. The in-sample evaluation of models

The four DCC models described in Section 2 were considered:

(1) The reference model is the DCC model of Engle (2002) summarized by equations (1)–(6), where parameters are estimated based only on closing prices.

(2) The DCC model of Tse and Tsui (2002) (Eqs. (11)–(14) with (5)–(6) for the GARCH model), i.e., a natural competitor of Engle's model, it was the basis for the construction of the new model. From the point of view of this paper, this model is not significantly different from the previous one and its parameters are also estimated with the use of only closing prices.

Table 1
The in-sample evaluation of the models and estimates of conditional variances and covariances for Monte Carlo replicates.

Criterion	DCC Engle		DCC Tse, Tsui		Range DCC		Co-range DCC	
ln L	−3,902.897		−3,903.091		−3,870.911		−3,867.568	
LF_1	9.200		9.195		6.940		6.726	
LF_2	0.193		0.220		0.192		0.168	
	Var	Cov	Var	Cov	Var	Cov	Var	Cov
MSE	1.127	0.178	1.127	0.178	0.813	0.144	0.813	0.133
MAE	0.560	0.212	0.560	0.212	0.459	0.193	0.459	0.184
R^2	0.340	0.281	0.340	0.285	0.578	0.428	0.578	0.440

Ln L is the logarithm of the likelihood function, LF_1 and LF_2 are the squared Frobenius loss functions for conditional covariance and correlation matrices (Eqs. (42)–(43)), the loss functions *MSE*, *MAE* and R^2 are calculated separately for variances and covariances of returns.

(3) The range-based DCC model of Chou et al. (2009) (Eqs. (21)–(24)). In this specification the CARR model (Eqs. (16)–(18)) is applied in the DCC model of Engle (2002) instead of the univariate GARCH model. It means that low and high prices are used only in the first stage of estimation.
(4) The proposed co-range DCC model (Eqs. (32)–(34) with (16)–(18) for the CARR model), where parameters are estimated based on low, high and closing prices.

In the co-range DCC model the Parkinson estimator Eq. (35)) was applied to calculate variances in formula ((31) and variances used in the correlation matrix $\mathbf{\Phi}_{t-1}$ in formula (33). It was also used and advocated by Brunetti and Lildholdt (2002) and Brandt and Diebold (2006). The main conclusions of our study do not change, however, for other estimators, such as, for instance, Garman and Klass (1980) or Rogers and Satchell (1991).

The parameters of all four analysed DCC models were estimated for each repetition in the Monte Carlo simulation using the maximum likelihood method. It means that all models were estimated 1,000 times. Due to large number of parameters it was not feasible to estimate the parameters of the true VECH model. First, we compared the quality of the models based on the three different criteria: the likelihood function and two squared Frobenius loss functions. Estimation of parameters of the GARCH and CARR models is based on different kind of data, i.e., on closing prices and range data for the former and latter models, respectively. However, for the range-based DCC and co-range DCC models, which use the CARR model, it is possible to calculate the likelihood function based on the scaled conditional price range according to formulas (26) and (37). Owing to this it is possible to evaluate all the DCC models based on the whole likelihood function including volatility and correlation parts.

Additionally, following Laurent et al. (2013) (see also de Almeida et al. 2018) the consistent squared Frobenius loss functions were used to evaluate the performance of the models for estimating conditional covariance and correlation matrices:

$$LF_1 = (1/n)\sum_{t=1}^{n} \mathrm{Tr}\left[(\widehat{\mathbf{cov}}_t - \mathbf{cov}_t)'(\widehat{\mathbf{cov}}_t - \mathbf{cov}_t)\right], \tag{42}$$

$$LF_2 = (1/n)\sum_{t=1}^{n} \mathrm{Tr}\left[(\widehat{\mathbf{cor}}_t - \mathbf{cor}_t)'(\widehat{\mathbf{cor}}_t - \mathbf{cor}_t)\right], \tag{43}$$

where Tr is a trace of a matrix, $\widehat{\mathbf{cov}}_t$ and $\widehat{\mathbf{cor}}_t$ are the estimated conditional covariance and correlation matrices, $\mathbf{cov}_t$ and $\mathbf{cor}_t$ are the real conditional covariance and correlation matrices (simulated from the VECH model).

The returns calculated as $r_t = 100\ln(p_t/p_{t-1})$, where p_t is the closing price at time t, were applied in the analysis. The averages from 1,000 series of simulations for the logarithms of the likelihood function and the squared Frobenius loss functions are given in Table 1. According to the likelihood function the co-range DCC model best described the dynamics of the 5 simulated series among the considered DCC models. The results for loss functions indicated that average losses, when estimating conditional covariances and correlations matrices, were the smallest also for this model. The second best model according to this criterion was the range-based DCC model. It performed much better, in comparison to the models based on closing prices, under the LF_1 function, i.e., for the estimation of the conditional covariance matrix.

From the practical point of view it is advisable to analyse also estimates of variances and covariances separately (in some applications only volatility of financial processes is used, in other cases the relationship between processes plays a key role). The joint analysis of the covariance matrix does not show whether the superiority of one model results from better modelling of variances, covariances or both. In this case the mean squared error (*MSE*), the mean absolute error (*MAE*) and additionally the coefficient of determination from the Mincer-Zarnowitz regression were calculated. These criteria are often used for an evaluation of volatility in empirical studies (see e.g. Poon and Granger, 2003; Patton and Sheppard, 2009; Violante and Laurent, 2012). Other loss functions were also considered, but yielded similar results. Thus, to save space in the whole paper the results only for these criteria were presented. The measures were computed as averages from 2,500,000 (1,000 repetitions × 500 observations in-sample × 5 time series) observations for variances and from 5,000,000 (1,000 × 500 × 10) observations for covariances. The results are also presented in Table 1. In the DCC models of Engle (2002) and Tse and Tsui (2002) the same standard univariate GARCH model was applied in the first stage of estimation, that is why the estimates of the measures *MSE, MAE* and R^2 were the same for variances. The same situation was for the range-based DCC and co-range DCC models, for which the CARR model was applied in the first stage of estimation.

All evaluation measures indicated that the CARR model described volatility of returns better than the GARCH model. This result was expected, but to the best of our knowledge, it has not been demonstrated in the literature yet, since all the studies have concentrated on out-of-sample forecasts. According to the loss functions *MSE, MAE* and R^2 estimates of covariances based on the co-range DCC were more accurate than estimates based on its competitors. The range-based DCC model was pointed out as the second best. The performance of the DCC models of Engle (2002) and Tse and Tsui (2002) was very similar.

3.2. The out-of-sample forecasting performance

For each competing DCC model 1,000 one-day-ahead out-of-sample forecasts of conditional covariance and correlation matrices were formulated. First, the performance of the forecasts based on the squared Frobenius loss functions defined in Eqs. (42)–(43) was evaluated, where the estimated conditional covariance and correlation matrices were replaced by its forecasts. In order to assess whether the differences between loss functions were statistically significant two different tests were applied: the test of superior predictive ability (SPA) of Hansen (2005) and the model confidence set (MCS) of Hansen et al. (2011). In the first approach, it was checked whether or not each of the models considered was outperformed significantly by any of the alternatives. In the second approach, the MCS contained the best forecasting models with a certain probability. The corresponding p-values for both tests are given in Table 2.

The results of the SPA test indicated that the only model, which was not outperformed significantly by any of the alternatives was the co-range DCC model. According to the results of the MCS test, only the co-range DCC model belonged to the MCS. The forecasting superiority of the co-range DCC model did not depend on the type of loss function.

Similarly like for the in-sample analysis, the performance of forecasts was also evaluated separately for conditional variances and covariances. The results for the three measures *MSE, MAE* and R^2 are reported in Table 3.

All evaluation measures indicated that variance forecasts based on the CARR model were more accurate than forecasts based on the GARCH model. The results for covariance forecasts were also unequivocal and clearly pointed out the co-range DCC model as the best forecasting model. Furthermore, the models based on closing prices were inferior to the models formulated with low and high prices.

Table 2
The out-of-sample evaluation of forecasts of covariance and correlation matrices for Monte Carlo replicates.

Model	Forecast evaluation criteria					
	LF_1	SPA p-value	MCS p-value	LF_2	SPA p-value	MCS p-value
DCC Engle	7.077	0.000	0.000	0.198	0.000	0.000
DCC Tse, Tsui	7.142	0.000	0.000	0.256	0.000	0.000
Range DCC	4.853	0.000	0.000	0.196	0.000	0.000
Co-range DCC	4.740	0.661	1.000*	0.172	0.547	1.000*

LF_1 and LF_2 are the squared Frobenius loss functions for conditional covariance and correlation matrices,
* indicates that models belong to the MCS with a confidence level of 0.90.

Table 3
The out-of-sample evaluation of the variance and covariance forecasts for Monte Carlo replicates.

Model	Forecast evaluation criteria						
	MSE	SPA p-value	MCS p-value	*MAE*	SPA p-value	MCS p-value	R^2
			Variance forecasts				
GARCH	0.837	0.000	0.000	0.562	0.000	0.000	0.361
CARR	0.544	0.507	1.000*	0.450	0.504	1.000*	0.603
			Covariance forecasts				
DCC Engle	0.133	0.000	0.000	0.209	0.000	0.000	0.267
DCC Tse, Tsui	0.136	0.000	0.000	0.214	0.000	0.000	0.261
Range DCC	0.101	0.000	0.000	0.188	0.000	0.000	0.430
Co-range DCC	0.095	0.522	1.000*	0.178	0.500	1.000*	0.458

* Indicates that models belong to the MCS with a confidence level of 0.90, R^2 is the coefficient of determination from the Mincer-Zarnowitz regression.

4. The analysis of currency rates in the Forex market

4.1. Data

The proposed model and its competitors were applied in the empirical study of the three most heavily traded currency pairs in the Forex market, namely: EUR/USD, USD/JPY and GBP/USD. First, a valuation of the models considered was performed for daily data for the eleven-year period from January 2, 2006, to December 30, 2016 (2,853 returns). It was a relatively long period, which included both very volatile periods like the global financial crisis of 2008, the European sovereign debt crisis and the 2016 Brexit referendum but also tranquil periods with low volatility.

The first two applied models, i.e., the DCC models of Engle (2002) and Tse and Tsui (2002) are formulated based on squared returns (the GARCH model; Eq. (6)) and return-based covariances (i.e., the product of daily returns; compare Eqs. (4) and (14)). The range-based DCC model is built on ranges (the CARR model; Eq. (18)) and return-based covariances (compare Eq. (24)). Whereas the co-range DCC model is formulated on ranges (the CARR model; Eq. (18)) but in the conditional covariance matrix the range-based variances and covariances are applied (Eq. (34)). It is vital to compare properties of those measures and confront them with properties of realized variances and covariances, which are used in the paper as the real values of daily variances and covariances in a forecasting evaluation. In this study the realized variance was estimated as the sum of squared 15-min returns, while the realized covariance was calculated as the sum of products of 15-min returns. The descriptive statistics for all considered series are presented in Table 4. It should be noticed that the range is not a proper measure of variance and it has to be scaled for this goal (see Sections 2.3 and 2.4).

Table 4
Summary statistics of proxies of volatility and covariance for selected exchange rates.

Series	Mean	Min	Max	SD	Skew	Kurt	LB(10)
Proxies of volatility							
EUR/USD							
Squared return	0.388	0.000	12.272	0.779	5.351	44.756	542
Range	0.930	0.097	4.628	0.510	1.932	6.513	5,314
Parkinson estimator	0.406	0.003	7.724	0.562	5.196	41.045	3,205
Realized variance	0.410	0.011	6.564	0.480	4.935	38.189	7,309
JPY/USD							
Squared return	0.448	0.000	29.675	1.147	10.059	179.258	264
Range	0.979	0.158	7.736	0.601	3.101	19.878	2,819
Parkinson estimator	0.476	0.009	21.582	0.944	11.617	204.784	797
Realized variance	0.488	0.017	17.782	0.810	9.901	152.522	2,037
GBP/USD							
Squared return	0.375	0.000	69.252	1.504	34.598	1 543.373	191
Range	0.897	0.126	12.707	0.570	5.162	73.211	6,466
Parkinson estimator	0.407	0.006	58.233	1.271	34.335	1 512.303	524
Realized variance	0.425	0.011	40.324	0.998	25.568	935.968	1,733
Proxies of covariance							
EUR/USD-JPY/USD							
Product of daily returns	0.089	−9.653	8.545	0.654	−1.154	64.028	202
Range-based covariance	0.089	−9.428	5.296	0.434	−4.689	138.781	875
Realized covariance	0.080	−6.237	4.766	0.311	−1.978	101.805	3,035
EUR/USD-GBP/USD							
Product of daily returns	0.232	−2.157	21.255	0.677	12.777	336.127	246
Range-based covariance	0.248	−0.984	18.326	0.531	16.942	498.987	1,529
Realized covariance	0.252	−0.078	11.302	0.397	10.810	231.065	6,580
JPY/USD-GBP/USD							
Product of daily returns	0.018	−31.449	8.555	0.866	−18.536	636.843	99
Range-based covariance	0.024	−21.137	4.034	0.621	−21.690	670.719	393
Realized covariance	0.025	−17.875	2.963	0.434	−26.482	1,038.938	1,022

Mean – the arithmetic mean, Min – minimum, Max – maximum, SD - standard deviation, Skew – skewness, Kurt - excess kurtosis, LB(10) – the Ljung-Box statistic for 10 lags.

The properties of proxies of volatility are commonly known. The estimator based on daily squared returns is unbiased, however not efficient, with much noise (see high values of standard deviation for all currencies). The values of the Ljung-Box statistic were much lower for this measure than for other proxies. It means that the autocorrelation for squared returns is weaker and it will be more difficult to forecast them in comparison to other volatility proxies. The properties of the proxies of covariance are less known but they are similar to the ones of variance. The estimator based on the product of daily returns is unbiased but not efficient. The values of standard deviation were higher and the values of the Ljung-Box statistic much lower for this proxy. The results indicated that range-based measures of both variance and covariance are more accurate and their properties are more similar to the ones of the realized variance and covariance that is why the application of them in volatility models should be very beneficial.

4.2. Comparison of models

The parameters of the four DCC models were estimated using the quasi-maximum likelihood method. The considered exchange rates were not cointegrated (according to the Johansen test) and there were no constant relations in the conditional means of returns (according to the VAR model), which is why there were only constants in the conditional mean equations of returns. The results of estimation are presented in Table 5.

Similarly like in Monte Carlo experiments the quality of the models was evaluated based on the three criteria: the likelihood function and two squared Frobenius loss functions. The results of these measures are given in Table 6. The Rivers and Vuong test (Rivers and Vuong, 2002) was performed, which allowed to verify the hypothesis that the likelihood functions of two non-nested competing DCC models are asymptotically equivalent. The Rivers and Vuong test is a generalization of the Vuong tests (Vuong, 1989), which can be applied to nonlinear models of time series. According to the results of the test the range-based DCC model and the co-range DCC model were significantly better than the DCC model of Engle (2002).

Table 5
The results of parameters estimation for the four DCC models for selected exchange rates.

Par.	DCC Engle		DCC Tse, Tsui		Par.	Range DCC		Co-range DCC	
	Estimate	SE	Estimate	SE		Estimate	SE	Estimate	SE
γ_{10}	0.003	0.010	0.003	0.010	γ_{10}	–	–	–	–
α_{10}	0.001	0.000	0.001	0.000	φ_{10}	0.005	0.002	0.005	0.002
α_{11}	0.037	0.005	0.037	0.005	φ_{11}	0.087	0.009	0.087	0.009
β_{11}	0.960	0.006	0.960	0.006	φ_{12}	0.907	0.010	0.907	0.010
γ_{20}	−0.014	0.011	−0.014	0.011	γ_{20}	–	–	–	–
α_{20}	0.007	0.003	0.007	0.003	φ_{20}	0.019	0.006	0.019	0.006
α_{21}	0.063	0.017	0.063	0.017	φ_{21}	0.134	0.018	0.134	0.018
β_{21}	0.923	0.020	0.923	0.020	φ_{22}	0.847	0.022	0.847	0.022
γ_{30}	−0.001	0.009	−0.001	0.009	γ_{30}	–	–	–	–
α_{30}	0.003	0.001	0.003	0.001	φ_{30}	0.006	0.002	0.006	0.002
α_{31}	0.070	0.026	0.070	0.026	φ_{31}	0.105	0.012	0.105	0.012
β_{31}	0.925	0.023	0.925	0.023	φ_{32}	0.888	0.012	0.888	0.012
θ_1	0.042	0.006	0.041	0.006	θ_1	0.044	0.006	0.109	0.015
θ_2	0.934	0.012	0.940	0.010	θ_2	0.935	0.011	0.880	0.017

The parameters γ_{10}, γ_{20}, γ_{30} are constants, α_{k0} α_{k1} β_{k1} Eq. (6)) and φ_{k0} φ_{k1} φ_{k2} (Eq. (18)) are the parameters of the univariate GARCH model and the CARR model respectively, $k = 1, 2, 3$ for EUR/USD, JPY/USD and GBP/USD respectively, θ_1, θ_2 are the parameters of the correlation part (Eqs. (2), ((13), (22) and (33)).

Table 6
The in-sample evaluation of the models and estimates of conditional covariance and correlation matrices for selected exchange rates.

Model	Model evaluation criteria							
	$\ln L$	*RV*	LF_1	SPA p-value	MCS p-value	LF_2	SPA p-value	MCS p-value
DCC Engle	−6,441.510	–	2.222	0.001	0.000	0.270	0.000	0.000
DCC Tse, Tsui	−6,453.542	−1.145	2.238	0.001	0.000	0.269	0.000	0.000
Range DCC	−6,394.816	2.744*	2.069	0.001	0.000	0.272	0.000	0.000
Co-range DCC	−6,378.404	3.191*	2.051	0.677	1.000*	0.214	0.622	1.000*

$\ln L$ is the logarithm of the likelihood function, *RV* is the Rivers-Vuong tests statistic for model selection and comparisons are made with the DCC model of Engle, LF_1 and LF_2 are the squared Frobenius loss functions for conditional covariance and correlation matrices, * for *RV* indicates that the null hypothesis is rejected at the 0.1 level, * for MCS indicates that models belong to the MCS with a confidence level of 0.90.

The squared Frobenius loss functions were calculated based on formulas (42)–(43), but in place of the real conditional covariance and correlation matrices, the realized covariance and correlation matrices were applied. All realized variances and covariances used in the study were calculated based on 15-min returns but the main results of the paper do not change for 5- or 30-min returns. According to the results of the SPA and MCS tests the most accurate estimates of covariance and correlation matrices were based on the co-range DCC model.

It is interesting to compare the estimates of parameters between the considered models. In the DCC models of Engle (2002) and Tse and Tsui (2002) the standard univariate GARCH model was used, that is why the estimates of parameters were different between these models only in the correlation part. The similar situation was for the range-based DCC and co-range DCC models, for which the CARR model was applied. The application of low and high prices changed the estimates of the parameters of the DCC models significantly. Specifically, the estimates of the parameters φ_{i1} were much higher in the CARR model than the estimates of the parameters α_{i1} in the GARCH model, while the estimates of the parameters φ_{i2} were lower than the estimates of the parameters β_{i1}. This empirical regularity has been already known in the literature (see Chou et al., 2009; Wu and Liang, 2011; Su and Wu, 2014). The differences between the standard GARCH model and the CARR model are natural and expected, because they describe different measures of volatility. The GARCH model describes the conditional variance of returns, while the CARR model describes the dynamics of the conditional mean of the price range.

Considerable differences were also present in the correlation component. The estimate of the parameter θ_1 in the co-range DCC model was more than two times higher, while the estimate of the parameter θ_2 was much lower compared with the estimates in other models. This is important for both modelling and forecasting the covariance of returns, because shocks in the previous period have a stronger impact on the current covariance, and thus, the proposed model based on the range-based covariance estimator has a faster response to changing market conditions. A slow response to abrupt changes in the market is widely cited as one of the greatest weaknesses of GARCH-type models formulated based on closing prices (e.g. Andersen et al., 2003; Hansen et al., 2012). It is also worth mentioning that the estimates of parameters in the correlation part were very similar in the remaining models (also in the range-based DCC model). It means that the key to the results obtained is the application of the range-based estimator of the covariance of returns.

4.3. The forecasting performance

In this section the forecasting performance of all considered DCC models was compared. Out-of sample one-day-ahead forecasts of covariance and correlation matrices were formulated based on the models, where parameters were estimated separately each day based on a rolling sample with a fixed size of 500 (approximately a two-year period; the first in-sample period was from January 3, 2006 to December 31, 2007). The evaluation of forecasts was performed for the nine-year period from January 2, 2008, to December 30, 2016.

First, we evaluated the forecasts of the whole covariance and correlation matrices based on the squared Frobenius loss functions defined in Eqs. (42)–(43), where the estimated conditional covariance and correlation matrices were replaced by its forecasts and the real conditional covariance and correlation matrices were replaced by realized covariance and correlation matrices. We evaluated whether the differences in the forecasting performance among the considered models were statistically significant by performing the SPA and MCS tests. The corresponding p-values are given in Table 7. The results of both tests indicated that forecasts of covariance and correlation matrices based on the co-range DCC model were the most accurate.

Similarly like in the simulation study, the analysis of the forecasting performance was also performed for variances and covariances separately. The results for variance forecasts are presented in Table 8.

The results of the SPA test indicated that the variance forecasts from the CARR model were significantly more accurate (at the 10% significance level) than the forecasts based on the standard GARCH model for the JPY/USD and GBP/USD exchange rates according to both loss functions and for the EUR/USD pair for the *MAE* measure. Similar conclusions came from the MCS test, with the difference that for both *MSE* and *MAE* criteria two models belonged to the MCS for EUR/USD and it was

Table 7
The out-of-sample evaluation of forecasts of covariance and correlation matrices for selected exchange rates.

Model	Forecast evaluation criteria					
	LF_1	SPA p-value	MCS p-value	LF_2	SPA p-value	MCS p-value
DCC Engle	2.870	0.010	0.045	0.285	0.000	0.000
DCC Tse, Tsui	2.862	0.010	0.026	0.291	0.000	0.000
Range DCC	2.751	0.010	0.002	0.382	0.000	0.000
Co-range DCC	2.536	1.000*	0.568	0.233	1.000*	0.527

LF_1 and LF_2 are the squared Frobenius loss functions for conditional covariance and correlation matrices, the realized covariance and the realized variance based on 15-min returns are applied.
* Indicates that models belong to the MCS with a confidence level of 0.90.

Table 8
The evaluation of the variance forecasts for selected exchange rates: the *MSE* and *MAE* criteria.

Model	Forecast evaluation criteria					
	MSE	SPA p-value	MCS p-value	*MAE*	SPA p-value	MCS p-value
			EUR/USD			
GARCH	0.168	0.115	0.207*	0.206	0.085	0.153*
CARR	0.160	0.885	1.000*	0.201	0.916	1.000*
			JPY/USD			
GARCH	0.631	0.012	0.012	0.313	0.000	0.000
CARR	0.595	0.519	1.000*	0.287	0.510	1.000*
			GBP/USD			
GARCH	1.190	0.066	0.060	0.230	0.0312	0.026
CARR	0.999	0.934	1.000*	0.207	0.534	1.000*

The realized variance is used as a proxy of the real values of variance and estimated as the sum of squared 15-min returns.
* Indicates that models belong to the MCS with a confidence level of 0.90.

Table 9
The evaluation of the covariance forecasts for selected exchange rates: the *MSE* and *MAE* criteria.

Model	Forecast evaluation criteria					
	MSE	SPA p-value	MCS p-value	*MAE*	SPA p-value	MCS p-value
			EUR/USD-JPY/USD			
DCC Engle	0.097	0.001	0.000	0.140	0.000	0.000
DCC Tse, Tsui	0.106	0.003	0.000	0.141	0.000	0.000
Range DCC	0.146	0.002	0.000	0.164	0.000	0.000
Co-range DCC	0.087	0.592	1.000*	0.126	0.518	1.000*
			EUR/USD-GBP/USD			
DCC Engle	0.123	0.079	0.095	0.131	0.018	0.002
DCC Tse, Tsui	0.120	0.098	0.122*	0.129	0.029	0.037
Range DCC	0.114	0.301	0.273*	0.130	0.000	0.002
Co-range DCC	0.112	0.970	1.000*	0.123	0.501	1.000*
			JPY/USD-GBP/USD			
DCC Engle	0.220	0.055	0.013	0.119	0.011	0.003
DCC Tse, Tsui	0.210	0.013	0.013	0.118	0.000	0.000
Range DCC	0.239	0.005	0.003	0.133	0.000	0.000
Co-range DCC	0.193	0.961	1.000*	0.107	0.550	1.000*

The realized covariance is used as the real values of covariance and estimated as the sum of products of 15-min returns.
* Indicates that models belong to the MCS with a confidence level of 0.90.

not possible to point out a significantly better model. The forecasting superiority of the CARR model over the GARCH model has already been documented in the literature (e.g. Chou, 2005, Chou and Wang, 2007).

Under the *MSE* measure, the lowest errors of the volatility forecasts were for the EUR/USD rate. Considerably higher errors were for the JPY/USD pair and the highest for the GBP/USD rate. Under the *MAE* criterion, which is a less sensitive measure to outliers, the errors were significantly lower for the GBP/USD pair. It indicated that the difficulty in volatility forecasting for this rate was mainly caused by outliers, which took place, e.g., after the Brexit vote.

The forecasting performance results for the covariance of returns are presented in Table 9. According to the results of the SPA test for the *MSE* and *MAE* criteria, except for one case, the forecasts from the co-range DCC model where the most precise (at the 10% significance level). Only for the *MSE* measure and the covariance between the EUR/USD and GBP/USD pairs there were two models, i.e., the co-range DCC and Range-based DCC, which were not outperformed significantly by any of the alternatives. The results of the MCS test were very similar and indicated the co-range DCC was the best forecasting model, with the same exception as for the SPA test. For the relation between the EUR/USD and GBP/USD pairs three models: the DCC of Tse and Tsui (2002), the range-based DCC and the co-range DCC belonged to the MCS and there was no evidence to reject the null hypothesis of equal predictive ability for those models.

Table 10
The evaluation of the variance and covariance forecasts for selected exchange rates: the coefficient of determination.

Model	Variances		
	EUR/USD	JPY/USD	GBP/USD
GARCH	0.367	0.193	0.097
CARR	0.429	0.238	0.192
Model	**Covariances**		
	EUR/USD-JPY/USD	EUR/USD-GBP/USD	JPY/USD-GBP/USD
DCC Engle	0.163	0.344	0.058
DCC Tse, Tsui	0.085	0.364	0.077
Range DCC	0.033	0.408	0.029
Co-range DCC	0.247	0.422	0.145

The realized covariance and the realized variance based on 15-min returns are used as the real values of covariance and variance, respectively.

It should be noted that the forecasting errors were significantly lower for the evaluation of covariance than for variance. Other loss functions were also considered in the study, but yielded similar results. Thus, to save space, Table 10 presents only the R^2 values from the Mincer-Zarnowitz regression.

According the coefficient of determination the forecasts of variances based on the CARR model were more accurate than the forecasts based on the GARCH model. For all covariances the highest R^2 values were obtained for the co-range DCC model and pointed again at this model as superior.

5. Conclusions

The DCC model is one of the most popular multivariate GARCH models used in modelling financial time series. Its parameters are most often estimated based solely on closing prices. However, for many financial assets low and high prices are commonly available and allow for much more efficient estimation of variances. In this study, we have proposed a new specification of the DCC model, called the co-range DCC model, where its parameters are estimated with the use of range-based variances and covariances. Such construction of the model permits for the usage of much more information about variation of returns and its relationships in estimation of its parameters. The range-based estimator of covariance and as a consequence the proposed DCC model require that the range of a portfolio return is given. This range can be calculated in some particular cases, for example, when cross rates of foreign exchange rates are given or when tick-by-tick data are available (the range-based estimator can be less sensitive than the realized covariance to some sources of the microstructure noise arising from the bid-ask spread and nonsynchronous trading; see Brandt and Diebold, 2006).

We have compared the proposed model with two return-based the DCC models, i.e., the DCC model of Engle (2002) and the DCC model of Tse and Tsui (2002), and also with the range-based DCC model of Chou et al. (2009). All these four models are dynamic conditional correlations models but differ in the specification of the conditional covariance or correlation matrices and the type of data used. We have conducted a simulation study by fitting the DCC models to the series generated by the general VECH model and have analyzed the implications of the usage of different specifications of models and different type of data for estimation and prediction of conditional covariance and correlation matrices. Additionally, we have also performed an analysis for most heavily traded currency pairs in the Forex market.

We have shown that the use of low and high prices can improve estimation of covariance and correlation matrices of returns and increase the accuracy of its forecasts based on the co-range DCC model, compared with using closing prices only. The proposed model has been superior in comparison to the return-based DCC models, but also to the range-based DCC model. Because the latter model has the same specification for variance as our model, it means that the main advantage of the suggested co-range DCC model comes from the application of the range-based covariance estimator. Main conclusions of the study are robust to the forecast evaluation criterion employed.

In future, this method could be extended to other multivariate GARCH models (it has already been done for the BEKK model in Fiszeder, 2018), as well as to other volatility models such as the multivariate stochastic volatility models. The range-based volatility models such as the double smooth transition conditional correlation CARR model of Chou and Cai (2009), the range-based copula models of Chiang and Wang (2011) and Wu and Liang (2011) or the range-based regime-switching dynamic conditional correlation model of Su and Wu (2014) could also benefit from applying the covariance estimator based on low and high prices.

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Appendix

The data generating process in Monte Carlo experiments was the VECH model in Eqs. (39)–(40) with parameters given in the following matrices:

$$\mathbf{A} = \begin{pmatrix} 0.081 & 0.02 & -0.01 & -0.012 & 0.015 & 0.02 & 0 & 0 & 0 & 0.005 & 0 & 0 & 0.011 & 0 & 0.01 \\ 0.02 & 0.053 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.019 & 0 & 0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0.017 & 0 & 0 & 0.037 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 \\ 0.018 & 0 & 0 & 0 & 0.044 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01 \\ 0.021 & 0.008 & 0 & 0 & 0 & 0.098 & -0.01 & -0.011 & 0.017 & 0.02 & 0 & 0 & 0.012 & 0 & 0.015 \\ 0 & 0 & 0 & 0 & 0 & 0.014 & 0.049 & 0 & 0 & 0.02 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.015 & 0 & 0.04 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0.045 & 0 & 0 & 0 & 0 & 0 & 0.012 \\ 0.018 & 0 & 0.01 & 0 & 0 & 0.017 & -0.009 & 0 & 0 & 0.1 & 0.01 & 0.011 & 0.015 & 0 & 0.019 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.015 & 0.039 & 0 & 0.023 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.02 & 0 & 0.045 & 0 & 0 & 0.019 \\ 0.02 & 0 & 0 & -0.01 & 0 & 0.024 & 0 & -0.01 & 0 & 0.014 & -0.01 & 0 & 0.074 & -0.011 & 0.022 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.019 & 0.039 & 0.014 \\ 0.017 & 0 & 0 & 0 & 0.007 & 0.02 & 0 & 0 & 0.009 & 0.015 & 0 & -0.01 & 0.02 & 0.01 & 0.07 \end{pmatrix}$$

$$\mathbf{B} = \mathrm{diag}(0.804\ \ 0.812\ \ 0.821\ \ 0.804\ \ 0.802\ \ 0.803\ \ 0.812\ \ 0.829\ \ 0.812\ \ 0.815\ \ 0.822\ \ 0.833\ \ 0.827\ \ 0.835\ \ 0.804),$$

$$\mathbf{\Sigma} = 10^{-4} \times ((1.345\ \ 0.326\ \ 0.517\ \ 0.330\ \ 0.423)'\ \ (0.326\ \ 1.863\ \ 0.642\ \ 0.383\ \ 0.42)'\ \ (0.517\ \ 0.642\ \ 1.911\ \ 0.506\ \ 0.702)'$$
$$(0.330\ \ 0.383\ \ 0.506\ \ 1.545\ \ 0.504)'\ \ (0.423\ \ 0.42\ \ 0.602\ \ 0.504\ \ 1.657)').$$

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