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Improving volatility forecasts: Evidence from range-based models

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ABSTRACT

Volatility models based on the daily high-low range have become increasingly popular. The high and low prices are easily available, yet the range contains very useful information about volatility. It has been established in the literature that range-based volatility models outperform standard volatility models based on closing prices. However, little is known about which range-based model performs the best. We therefore evaluate two range-based volatility models, i.e. CARR and Range-GARCH with the standard GARCH model and two asymmetric GARCH models, i.e., GJR and EGARCH, based on the Monte Carlo experiments and a wide sample of currencies and stock indices. For simulated time series, the range-based models outperform the standard GARCH model and asymmetric models, and the performance of the Range-GARCH model and the CARR model is similar. However, for real financial time series (six currency pairs and nine stock indices) the Range-GARCH model outperforms the standard GARCH, GJR, EGARCH, and CARR models, while ranking of the competing models is ambiguous. We argue that Range-GARCH is the best from the competing models.

1. Introduction

Volatility plays a crucial role in many areas of finance, including investments, risk management, hedging, assets valuation, and allocation but also in various economic applications, such as macroeconomic modelling. The first models for time-varying volatility were introduced by Engle (1982) and Bollerslev (1986). The GARCH (generalized autoregressive conditional heteroskedastic) model of Bollerslev (1986) is still the most popular volatility model.

The GARCH models are formulated based only on the data of closing prices. Meanwhile, more accurate estimates of variance can be constructed from daily low and high prices (Parkinson,1980; for an overview of range-based volatility estimators see Molnár, 2012), and this insight has led to more precise volatility models (see e.g. Chou, 2005; Brandt and Jones, 2006; Asai, 2013; Fiszeder and Perczak, 2016; Molnár, 2016; Xie, 2019; Fiszeder and Małecka, 2022; Fiszeder et al., 2023a; Fiszeder et al., 2023b). Daily low and high prices are almost always commonly available with closing prices for financial series. Therefore, their utilization in volatility models is very important from a practical viewpoint and quite easy to implement.

There is an agreement in the existing literature that range-based volatility models outperform models based on closing prices (see the reviews in Chou et al., 2015; Petropoulos et al., 2022). However, a comparison of range-based models is lacking in the literature. We therefore compare two range-based models, i.e. the CARR (conditional autoregressive range) model of Chou (2005) and the range-

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GARCH (RGARCH) model of Molnár (2016). We also compare these models with the standard GARCH model and two asymmetric models, i.e., the GJR (Glosten, Jagannathan, Runkle) model of Glosten et al. (1993) and the EGARCH (exponential GARCH) model of Nelson (1991). The GARCH, GJR, and EGARCH models belong to the most popular univariate volatility models formulated based on returns constructed on closing prices, while the CARR model is based on price ranges. The RGARCH model uses range data, but at the same time it is simple and can be easily applied.

We carry out Monte Carlo experiments by fitting the five volatility models formulated on closing or low and high prices to systems that are simulated by the stochastic volatility models. We then analyze the implications of the usage of different kinds of data for the estimation and prediction of conditional variance. Additionally, we perform an empirical analysis for a relatively large sample of currencies and stock indices.

This study has four main contributions. Firstly, we show that the way in which low and high prices are applied in the model is crucial. For currencies and stock indices, the performance of the CARR model is not generally significantly better than the performances of the standard GARCH, GJR, and EGARCH models, but at the same time the RGARCH model is significantly better than the standard GARCH, GJR, and EGARCH models.

Secondly, we compare forecasts from the two range-based models, i.e. CARR and RGARCH, and show that RGARCH is significantly more accurate for financial series and no worse than the CARR model for simulated series. These models have not been previously compared in the literature. Although, Fiszeder et al. (2019) analyzed the CARR and RGARCH models, they applied the multivariate DCC-CARR and DCC-RGARCH models.

Thirdly, we analyze the influence of the level of currencies and stock indices' volatility on the performance of the models. We show that for very high volatility, the CARR model performs better than the standard GARCH model. This result could explain the relatively good behavior of the CARR model in other studies (Chou, 2005; Chou and Wang, 2007; Liu and Wu, 2017; Fiszeder and Fałdziński, 2019). However, the RGARCH model outperforms not only the GARCH models, but also the CARR model during periods of extreme high market volatility. This conclusion is important, since such periods are often associated with market turmoil and high market uncertainty.

Fourthly, we perform Monte Carlo experiments and analyze the finite sample properties of the predictions of the conditional variances that are obtained by fitting the GARCH, GJR EGARCH, CARR, and RGARCH models to returns generated by the stochastic volatility (SV) model. According to our knowledge, it is the first comparison of daily range-based models on simulated data generated by the SV model for both daily and intraday data. In earlier studies based on a simulation experiment with the SV model, it was assumed that variance is constant during the day (e.g. Alizadeh et al., 2002; Molnár, 2016).

The rest of the paper is organized in the following way. Section 2 describes the applied models, i.e. GARCH, GJR, EGARCH, CARR, and RGARCH. Section 3 compares the models by carrying out Monte Carlo experiments to analyze the effects of their specifications on the estimation and forecasting of conditional variance. In Section 4, the performance of the models is compared for six currency pairs and nine stock indices. Section 5 provides conclusions.

2. Theoretical background

2.1. GARCH models

The GARCH model of Bollerslev (1986) is the most popular univariate volatility model and is based solely on closing prices. We apply this model in the paper as a benchmark for comparison with the range-based models. The GARCH model describes the dynamics of the conditional variance of returns.

Let us assume that the ε_t is the univariate innovation process for the conditional mean (or in a particular case the return process) and can be written as:

$$\varepsilon_t|\psi_{t-1} \sim N(0, h_t), \tag{1}$$

where ψ_{t-1} is the set of all information available at time $t-1$, N is the conditional normal distribution, h_t is the conditional variance. The GARCH $(1,1)$ model is the most frequently used model in empirical studies. It is presented as:

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \tag{2}$$

where $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$.

The parameters of the GARCH model can be estimated by the quasi-maximum likelihood (QML) method. The log-likelihood function can be written as:

$$L(\varsigma) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{n}\left(\ln h_t + \frac{\varepsilon_t^2}{h_t}\right), \tag{3}$$

where ς is a vector containing unknown parameters of the model, n is the number of daily observations used in estimation.

The estimates obtained by the QML method are consistent and asymptotically normal (see Weiss, 1986; Bollerslev and Wooldridge, 1992; Straumann, 2005).

Hansen and Lunde (2005) found that, for exchange rate data, more sophisticated GARCH models do not outperform the simple GARCH $(1,1)$ model. Nevertheless, we select two asymmetric GARCH models, i.e., the GJR and EGARCH models and compare them with the competing models. These two models are based solely on closing prices and belong to the most popular extensions of the standard GARCH model.

The GJR $(1,1)$ model introduced by Glosten et al. (1993) is given as:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \theta_1 I_{t-1} \varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \tag{4}$$

where I_{t-1} is a dummy variable which satisfies $I_{t-1} = 1$ when $\varepsilon_{t-1} \leq 0$ and $I_{t-1} = 0$ when $\varepsilon_{t-1} > 0$, and the parameters meet the following requirements: $\alpha_0 > 0$, $\alpha_1 \geq 0$, $\alpha_1 + \theta_1 \geq 0$, $\beta_1 \geq 0$.

The EGARCH $(1,1)$ model of Nelson (1991) can be specified as:

$$\ln h_t = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}| + \theta_1 \varepsilon_{t-1}}{h_{t-1}^{0.5}} + \beta_1 \ln h_{t-1} \tag{5}$$

The logarithmic form of the conditional variance means that it is not necessary to introduce any restrictions on parameters to ensure the positivity of the conditional variance.

The parameters of the GJR and EGARCH models can be estimated by the QML method and the likelihood function is similar to the standard GARCH model.

2.2. CARR model

The CARR model of Chou (2005) is a popular univariate volatility model based on price range. It is the main competitor of the RGARCH model in the class of range-based models.

When H_t and L_t are high and low prices over a day, respectively, and the observed price range is given as $R_t = ln(H_t) - ln(L_t)$, the CARR $(1,1)$ model can be written as:

$$R_t = \lambda_t u_t, \tag{6}$$

$$u_t|\psi_{t-1} \sim \exp(.), \tag{7}$$

$$\lambda_t = \alpha_0 + \alpha_1 R_{t-1} + \beta_1 \lambda_{t-1}, \tag{8}$$

where $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$, λ_t is the conditional mean of the range and u_t is the disturbance term, $\exp(.)$ is the exponential distribution with unit mean. The exponential distribution is a natural choice for the conditional distribution of u_t because it takes positive values.

The CARR model describes the dynamics of the conditional mean of the price range, that is why in order to estimate values of the conditional standard deviation of returns the conditional price range has to be scaled according to the formula: $\lambda_t^* = adj\lambda_t$ where $adj = \overline{\sigma}/\overline{\lambda}$, $\overline{\sigma} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\varepsilon_t - \overline{\varepsilon})^2}$, $\overline{\varepsilon} = \frac{1}{n}\sum_{t=1}^{n}\varepsilon_t$, the ε_t term is described in equation (1), $\overline{\lambda} = \frac{1}{n}\sum_{t=1}^{n}\lambda_t$. It means that the scaling factor adj is estimated as the quotient of the unconditional standard deviation of returns by the sample mean of the conditional range.

The parameters of the CARR model can be estimated by the QML method. The log-likelihood function can be described as:

$$L(\varsigma) = -\sum_{t=1}^{n}\left(\ln\lambda_t + \frac{R_t}{\lambda_t}\right). \tag{9}$$

The estimators obtained by the QML method are consistent (see Engle and Russell, 1998; Engle, 2002; Chou, 2005).

In order to compare the values of the log-likelihood function between the CARR and GARCH models, it is possible to calculate the likelihood function of the CARR model based on the scaled conditional price range according to the following formula:

$$L(\varsigma) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{n}\left(2\ln\lambda_t^* + \frac{\varepsilon_t^2}{\lambda_t^{*2}}\right), \tag{10}$$

where λ_t^* is the scaled conditional price range, and ε_t is the same as in equation (3).

2.3. Range-GARCH model

The Range-GARCH model was introduced by Molnár (2016). It is, like the CARR model, a range-based model, but its specification is more similar to the standard GARCH model. The RGARCH $(1,1)$ model can be formulated as:

$$\varepsilon_t|\psi_{t-1} \sim N(0, h_t), \tag{11}$$

$$h_t = \alpha_0 + \alpha_1 \sigma_{P t-1}^2 + \beta_1 h_{t-1}, \tag{12}$$

where $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$, $\sigma_{P,t}^2$ is the Parkinson estimator (Parkinson, 1980) given as $\sigma_{P,t}^2 = [ln(H_t/L_t)]^2/(4\ln 2)$.

The parameters of the RGARCH model can be estimated by the QML method and the likelihood function is similar to the standard GARCH model. The RGARCH model, like the standard GARCH model based on closing returns, describes the dynamics of conditional variance of returns.

The advantage of both the RGARCH and CARR models over the GARCH model comes from the usage of additional information about quotations during a day. The standard GARCH model is based on returns of closing prices. It means that the path of price during a day is totally ignored when volatility is estimated by such a model. Especially in turbulent days with drops and recoveries in the markets, the traditional close-to-close volatility indicates a low level, while the daily price range shows correctly that the volatility is high. Both the RGARCH and CARR model are based on high-low range, that is why they can estimate volatility more accurately. The CARR model describes the price range directly, whereas in the RGARCH model, the Parkinson estimator $\sigma^2_{P,t-1}$ is implemented in place of ε^2_{t-1}. The Parkinson estimator is several times more efficient at estimating variance than the squared closing return (it is 4.9 times more efficient on the assumption of Brownian motion, see Parkinson, 1980).

In the place of the Parkinson estimator, other range-based estimators like Garman-Klass (Garman and Klass, 1980) or Rogers-Satchell (Rogers and Satchell, 1991) can be applied in the RGARCH model. Molnár (2016) additionally implemented the Garman-Klass estimator in his study of stocks and stock indices and found that this estimator does not improve results. Moreover, opening price, which is used in Garman-Klass estimator, is sensitive to microstructure effects associated with low liquidity during the start of quotations. On the other hand, the Rogers-Satchell estimator has another drawback, namely it can take zero value despite big changes during a day. It happens when the opening price is equal to the low price, and the closing price is equal to the high price or vice versa, i.e., the opening price is equal to the high price and the closing price is equal to the low price. For these reasons, we apply only the Parkinson estimator in the RGARCH model.

3. Monte Carlo simulation

First, we check the performance of the considered models on simulated data. For this purpose, we conduct Monte Carlo experiments and analyze the finite sample properties of the predictions of conditional variances that are obtained by fitting the GARCH, GJR, EGARCH, CARR, and RGARCH models to returns generated by the stochastic volatility model. The SV model is chosen as the data generating process because of its flexibility. The SV model assumes two error processes, while the GARCH model allows for a single error term. The volatility under a stochastic volatility model is a random variable, in contrast to the GARCH model in which the conditional variance is a deterministic function of the model parameters and past data. This implies that the SV model can be more flexible than the GARCH model in fitting the data. Carnero et al. (2004) find that in the GARCH model, the parameters explaining persistence and kurtosis are closely linked, whereas these features can be modelled independently in the SV model, so the latter can better represent the empirical regularities often observed in financial time series (see also Danielsson, 1994; Kim et al., 1998).

Daily volatility is simulated by a stochastic volatility model similar to the one described by Alizadeh et al. (2002):

$$\ln\sigma_t = \ln\overline{\sigma} + \rho_H(\ln\sigma_{t-1} - \ln\overline{\sigma}) + \beta\varepsilon_{t-1}\sqrt{H}, \quad \text{for } t = 1, 2, \cdots, n_d, \tag{13}$$

where n_d is the number of days and ε_t is $N(0,1)$ innovation, i.e. following the normal distribution with zero mean and unit variance.

We start with the same values as Alizadeh et al. (2002), i.e. $H = 1/257$ and the first set of values (1) $\ln\overline{\sigma} = -2.5$, $\beta = 0.75$, $\rho_H = 0.985$. These volatility dynamics are broadly consistent with the literature on stochastic volatility. To check if the results are sensitive to accepted parameters values we consider also additional six sets of values: (2) $\ln\overline{\sigma} = -2$, $\beta = 0.75$, $\rho_H = 0.985$, (3) $\ln\overline{\sigma} = -3$, $\beta = 0.75$, $\rho_H = 0.985$, (4) $\ln\overline{\sigma} = -2.5$, $\beta = 0.7$, $\rho_H = 0.985$, (5) $\ln\overline{\sigma} = -2.5$, $\beta = 0.8$, $\rho_H = 0.985$, (6) $\ln\overline{\sigma} = -2.5$, $\beta = 0.75$, $\rho_H = 0.98$, (7) $\ln\overline{\sigma} = -2.5$, $\beta = 0.75$, $\rho_H = 0.99$.

For each day t, we simulate intraday prices with the following formulae:

$$s_{it} = s_{i-1t} + \sigma_{it}\varepsilon_{s,it}\sqrt{\Delta t}, \tag{14}$$

$$\ln\sigma_{it} = \ln\sigma_t + \rho_H(\ln\sigma_{i-1t} - \ln\sigma_t) + \beta\varepsilon_{\nu,i-1t}\sqrt{H}, \quad \text{for } i = 1, 2, \cdots, n_i, \tag{15}$$

where s_{it} is the logarithm of price, $\varepsilon_{s,it}$ and $\varepsilon_{\nu,it}$ are independent $N(0,1)$ innovations, $H = 1/257$, $dt = H/n_i$.

The discrete time increment Δt, a small fraction of the discrete sampling interval H, approximates the continuous time dt. We assume the same sets of parameters as in equation (13) except $\ln\overline{\sigma}$, which equals the current daily value $\ln\sigma_t$. As starting values for s_{it} and $\ln\sigma_{it}$, we take the last values from the previous day, i.e., s_{n_it-1} and $\ln\sigma_{n_it-1}$, respectively. We simulate 3,610 daily volatilities by equation (13) (this value is close to the average length of daily data of currencies and stock indices analyzed in the next section), where each daily price path is generated by $n_i = 100,000$ intraday price quotes based on equations (14) and (15). Observations from 1 to 3600 are applied for the in-sample analysis, while the last 10, i.e., from 3601 to 3610 are used for the evaluation of the out-of-sample performance for forecasts of variance of returns. More precisely, the 3,601st observation is used for the evaluation of the one-day-ahead forecasts, the 3,605th observation is used for the evaluation of five-day-ahead (i.e. for the fifth day ahead) forecasts, and the 3,610th observation is used for the evaluation of ten-day-ahead (i.e. for the tenth day ahead) forecasts. We apply 1,000 repetitions in the Monte Carlo simulations. Based on intraday series daily low, high, and closing prices are recorded and the percentage logarithmic returns and ranges, i.e. multiplied by 100, are used to estimate the parameters of the competing models.

It needs to be emphasized that we tried to simulate the data in such a way that it should not favor either of the competing models directly i.e. GARCH, GJR, EGARCH, CARR or RGARCH. Simulated data have already been applied to evaluate daily range-based variance estimators (e.g. Shu and Zhang, 2006; Buescu, et al., 2013), and daily range-based volatility models (e.g. Alizadeh et al., 2002; Molnár, 2016; Fiszeder and Fałdziński, 2019), but in these studies it was assumed that intraday prices were generated by the

Table 1
The results of the parameter estimates for the GARCH, CARR, RGARCH, GJR and EGARCH models for Monte Carlo simulation.

Parameters of simul. series	GARCH				CARR			RGARCH			
	γ_0	α_0	α_1	β_1	α_0	α_1	β_1	γ_0	α_0	α_1	β_1
$\ln\bar{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.001	0.005	0.052	0.933	0.0015	0.134	0.850	0.001	0.005	0.136	0.851
	(0.010)	(0.002)	(0.009)	(0.012)	(0.004)	(0.010)	(0.012)	(0.010)	(0.003)	(0.027)	(0.030)
$\ln\bar{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.002	0.014	0.052	0.933	0.024	0.134	0.850	0.002	0.014	0.136	0.851
	(0.016)	(0.005)	(0.009)	(0.012)	(0.006)	(0.010)	(0.012)	(0.016)	(0.008)	(0.027)	(0.030)
$\ln\bar{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.001	0.002	0.052	0.933	0.009	0.132	0.852	0.001	0.002	0.130	0.857
	(0.012)	(0.001)	(0.009)	(0.012)	(0.002)	(0.012)	(0.012)	(0.006)	(0.001)	(0.025)	(0.028)
$\ln\bar{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.001	0.005	0.048	0.937	0.014	0.124	0.860	0.001	0.005	0.125	0.861
	(0.010)	(0.002)	(0.009)	(0.012)	(0.004)	(0.010)	(0.012)	(0.010)	(0.003)	(0.026)	(0.029)
$\ln\bar{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.001	0.005	0.057	0.929	0.014	0.142	0.843	0.001	0.005	0.140	0.848
	(0.010)	(0.002)	(0.009)	(0.012)	(0.003)	(0.001)	(0.012)	(0.001)	(0.003)	(0.026)	(0.028)
$\ln\bar{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.001	0.007	0.048	0.931	0.019	0.129	0.849	0.001	0.007	0.131	0.849
	(0.010)	(0.003)	(0.009)	(0.015)	(0.004)	(0.010)	(0.013)	(0.010)	(0.004)	(0.027)	(0.032)
$\ln\bar{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.001	0.004	0.052	0.939	0.010	0.127	0.862	0.001	0.003	0.125	0.867
	(0.01)	(0.001)	(0.008)	(0.010)	(0.011)	(0.010)	(0.011)	(0.010)	(0.002)	(0.024)	(0.025)

Parameters of simul. series	GJR					EGARCH				
	γ_0	α_0	α_1	β_1	θ_1	γ_0	α_0	α_1	β_1	θ_1
$\ln\bar{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.001	0.005	0.052	0.934	0.000	0.001	−0.019	0.113	0.983	0.001
	(0.010)	(0.001)	(0.012)	(0.012)	(0.002)	(0.010)	(0.007)	(0.018)	(0.006)	(0.010)
$\ln\bar{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.002	0.014	0.052	0.933	0.000	0.002	−0.001	0.113	0.981	0.001
	(0.016)	(0.005)	(0.011)	(0.012)	(0.013)	(0.016)	(0.002)	(0.018)	(0.018)	(0.010)
$\ln\bar{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.001	0.002	0.051	0.934	0.001	0.001	−0.035	0.112	0.983	0.000
	(0.006)	(0.001)	(0.011)	(0.012)	(0.013)	(0.006)	(0.012)	(0.017)	(0.006)	(0.010)
$\ln\bar{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.001	0.005	0.047	0.937	0.000	0.001	−0.020	0.103	0.982	0.001
	(0.010)	(0.002)	(0.011)	(0.012)	(0.013)	(0.010)	(0.007)	(0.017)	(0.006)	(0.017)
$\ln\bar{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.001	0.005	0.056	0.930	0.001	0.001	−0.018	0.121	0.983	0.000
	(0.010)	(0.002)	(0.011)	(0.012)	(0.014)	(0.010)	(0.007)	(0.018)	(0.006)	(0.010)
$\ln\bar{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.001	0.007	0.048	0.931	0.000	0.001	−0.026	0.104	0.977	0.001
	(0.010)	(0.003)	(0.011)	(0.015)	(0.013)	(0.010)	(0.010)	(0.019)	(0.008)	(0.010)
$\ln\bar{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.001	0.004	0.051	0.939	0.001	0.001	−0.012	0.112	0.988	0.000
	(0.010)	(0.001)	(0.010)	(0.010)	(0.013)	(0.010)	(0.005)	(0.016)	(0.004)	(0.010)

The γ_0 parameter is the constant in the mean equation; α_0, α_1, β_1 are the parameters of the GARCH model (equation (2)), the CARR (equation (8)), and the RGARCH model (equation (12)); α_0 α_1 $\beta_{1,}$ θ_1 are the parameters of the GJR model (equation (4)) and the EGARCH model (equation (5)). The means of the estimated parameters and their standard deviations (in parentheses) are reported.

geometric Brownian motion. Such an assumption means that intraday returns have constant volatility. In contrast, in this study we assume that for simulated data, volatility is time-varying both for daily and intraday data.

3.1. The in-sample evaluation of models

The five volatility models described in Section 2 are considered:

1) The standard GARCH model of Bollerslev (1986) summarized by equations (1)-(2). Its parameters are estimated based on closing prices. It is a benchmark model for the range-based models;
2) The GJR model of Glosten et al. (1993) given by equations (1) and (4). Its parameters are estimated based on closing prices. It is a benchmark model for the range-based models;
3) The EGARCH model of Nelson (1991) defined by equations (1) and (5). Its parameters are estimated based on closing prices. It is a benchmark model for the range-based models;
4) The CARR model of Chou (2005) described by equations (6), (7), and (8). It is based on price range data;
5) The RGARCH model of Molnár (2016) given by equations (11) and (12). It is a competitor of the CARR model and is also the range-based volatility model.

The parameters of all models are estimated for each repetition of the Monte Carlo simulation using the QML method. This means that all models are estimated 1,000 times. In Table 1, we present the means of estimated parameters and their standard deviations for all sets of parameter values. The estimates of parameter β_1 are much higher and the estimates of parameter α_1 much lower in the standard GARCH model in comparison to the CARR and RGARCH models. The range-based volatility proxies are less noisy than squared returns, which is why more weight is put on the new information.

In the standard GARCH, GJR, and EGARCH models, both the conditional variance and the likelihood function are obtained only with returns of closing prices. In the RGARCH model, the Parkinson estimator with the high-low range is used as an explanatory variable in the conditional variance, but the likelihood function is formulated based on returns of closing prices. Interestingly, in the

Table 2
The in-sample evaluation of the models for Monte Carlo simulation: the logarithm of the likelihood function and the Rivers-Vuong test.

Parameters of simulated series	Log-likelihood					Percentage of rejections by RV test					
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH vs CARR	GARCH vs RGARCH	CARR vs RGARCH	RGARCH vs CARR	GJR vs RGARCH	EGARCH vs RGARCH
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	−2566.02	−2525.40	**−2524.84**	−2565.32	−2565.04	100 %	100 %	4.8 %	2.6 %	100 %	100 %
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	−4066.83	−4026.21	**−4025.64**	−4066.13	−4125.47	100 %	100 %	4.8 %	2.8 %	100 %	100 %
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	−1044.39	−1005.32	**−1004.29**	−1043.82	−1046.28	100 %	100 %	5.2 %	1.0 %	100 %	100 %
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	−2525.45	−2488.69	**−2488.47**	−2524.77	−2527.98	100 %	100 %	3.3 %	3.0 %	100 %	100 %
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	−2524.87	−2482.15	**−2480.64**	−2524.28	−2526.78	100 %	100 %	9.0 %	0.9 %	100 %	100 %
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	−2528.75	**−2492.24**	−2492.62	−2528.05	−2530.05	100 %	100 %	1.2 %	5.4 %	100 %	100 %
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	−2614.20	−2575.70	**−2573.05**	−2613.63	−2616.66	100 %	100 %	18.1 %	0.3 %	100 %	100 %

The means of the logarithm of the likelihood function are reported; the highest values are marked in bold. The percentage of rejections of the null hypothesis (at 10% significance level) by the Rivers-Vuong test indicates the cases for which the second model is superior to the first model in the pair.

Table 3
The out-of-sample evaluation of the one-day-ahead variance forecasts for Monte Carlo simulation: the real variance used for the MAE criterion.

Parameters of simulated series	MAE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.171	0.164	**0.159**	0.171	0.170	0.000	0.000	0.567	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.466	0.447	**0.434**	0.467	0.464	0.000	0.000	0.595	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.067	0.064	**0.063**	0.067	0.067	0.000	0.000	0.551	0.000	0.000	0.000	0.001	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.157	0.149	**0.145**	0.157	0.166	0.000	0.000	0.557	0.000	0.041	0.001	0.001	1.000*	0.001	0.001
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.175	0.167	**0.163**	0.176	0.175	0.000	0.000	0.564	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.153	0.145	**0.143**	0.153	0.153	0.000	0.000	0.582	0.000	0.000	0.000	0.001	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.216	0.212	**0.205**	0.216	0.217	0.000	0.000	0.639	0.000	0.000	0.000	0.000	1.000*	0.000	0.000

The lowest values of the MAE measure are in bold; p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing which model is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly; * indicates that model belongs to the model confidence set with a confidence level of 0.90.

Table 4
The out-of-sample evaluation of the one-day-ahead variance forecasts for Monte Carlo simulation: the real variance used for the MSE criterion.

Parameters of simulated series	MSE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.060	0.055	**0.054**	0.060	0.059	0.000	0.187	0.818	0.000	0.001	0.000	0.369*	1.000*	0.000	0.001
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.446	0.404	**0.401**	0.447	0.439	0.000	0.204	0.798	0.000	0.001	0.000	0.413*	1.000*	0.000	0.001
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.0123	**0.011**	**0.011**	0.013	0.013	0.007	0.491	0.297	0.008	0.018	0.008	1.000*	0.587*	0.007	0.013
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.047	0.043	**0.042**	0.047	0.146	0.000	0.518	0.936	0.000	0.076	0.080	0.443*	1.000*	0.080	0.080
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.084	**0.075**	**0.075**	0.084	0.084	0.001	0.443	0.558	0.005	0.016	0.006	0.866*	1.000*	0.005	0.007
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.045	**0.040**	**0.040**	0.045	0.045	0.000	0.659	0.341	0.000	0.000	0.000	1.000*	0.706*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.151	**0.141**	**0.141**	0.151	0.153	0.018	0.483	0.517	0.015	0.017	0.012	0.964*	1.000*	0.012	0.012

The lowest values of the MSE measure are in bold; p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing which model is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly; * indicates that model belongs to the model confidence set with a confidence level of 0.90.

CARR model, both the conditional range and the likelihood function are calculated only with price ranges. That is why the value of the likelihood function of the CARR model is not directly comparable with values of two other models. However, it is possible to compute the likelihood function based on the scaled conditional price range according to formula (10). The means of the logarithm of the likelihood function based on 1,000 repetitions are given in Table 2.

For six out of seven sets of parameters, the means of the logarithm of the likelihood function are highest for the RGARCH model. We compare the five models and assess whether the differences between the values of the likelihood function are statistically significant according to the Rivers and Vuong (2002) test. It allows verification of the hypothesis that the likelihood functions of two non-nested competing models are asymptotically equivalent. In Table 2, we present the percentage of rejections of the null hypothesis (at 10% significance level) for pairs of models. In all cases, both the CARR and RGARCH models are superior to the standard GARCH model. The RGARCH model is also always superior to the GJR and EGARCH models. On the other hand, in most cases it is not possible to indicate a better model between the CARR and RGARCH models, although the percentages pointing to the RGARCH model are most often considerably higher, especially when the persistence of volatility is high (for $\rho_H = 0.99$ in simulated series of the SV model in equation (13)).

3.2. The out-of-sample forecasting performance

For each competing model, 1,000 one-day-ahead, five-day-ahead, and ten-day-ahead out-of-sample forecasts of conditional variance are formulated. The evaluation of forecasts is performed on the basis of two basic measures, namely the mean squared error (MSE) and the mean absolute error (MAE). The MSE is the most frequently used criterion in forecasting studies. It can be written as:

$$\text{MSE} = \frac{1}{m}\sum_{t=1}^{m}\left(\sigma_{R,t}^2 - \sigma_{F,t}^2\right)^2, \tag{16}$$

where $\sigma_{R,t}^2$ is the true daily variance of returns calculated as $(\exp(\ln\sigma_t))^2$, $\ln\sigma_t$ is given by formula (13), $\sigma_{F,t}^2$ is the forecast of variance of returns at time t, and m is the number of forecasts.

The MSE is robust to the use of a noisy volatility proxy (it yields the same ranking of competing forecasts using an unbiased volatility proxy, see Hansen and Lunde, 2006 and Patton, 2011).

The MAE is less sensitive to outliers, which is very important when evaluating extraordinary events. It is given as:

$$\text{MAE} = \frac{1}{m}\sum_{t=1}^{m}\left|\sigma_{R,t}^2 - \sigma_{F,t}^2\right|. \tag{17}$$

Two tests were applied to evaluate the significance of the results, namely the test of superior predictive ability (SPA) of Hansen (2005) and the model confidence set (MCS) of Hansen et al. (2011). In the first test, each of the considered models is checked to determine if it could be outperformed significantly by any of the alternatives. The MCS procedure is used for all models, i.e., GARCH, GJR, EGARCH, CARR, and RGARCH, jointly, and the model confidence set contains the best forecasting models with a certain probability. The results for the one-day-ahead forecasts are presented in Tables 3 and 4 for the MAE and MSE criteria, respectively.

According to the MAE measure, both tests clearly indicate the RGARCH model is superior. Moreover, the forecasts based on the CARR model are more accurate than the forecasts from the standard GARCH, GJR, and EGARCH models.

The results for the MSE loss function are less explicit. The lowest values of this measure are again for the RGARCH model, but for four sets of parameters of simulated series, the MSE values are equal for the RGARCH and CARR models. The advantage of the RGARCH model is not statistically significant according to the applied tests. The forecasts based on both the CARR and RGARCH are more precise than the forecasts based on the standard GARCH, GJR, and EGARCH models, but at the same time there is no significant difference between the forecasts based on the CARR and RGARCH models.

The results for the five-day-ahead forecasts are presented in the Appendix in Tables A1 and A2 for the MAE and MSE criteria, respectively, and the results for the ten-day ahead forecasts are given in Tables A3 and A4 for the MAE and MSE measures, respectively. For almost all sets of parameters of simulated series, forecasts based on the RGARCH model are significantly more accurate than forecasts based on the standard GARCH, GJR, EGARCH, and CARR models. This indicates that the advantage of the RGARCH model is more visible for the five- and ten-day-ahead forecasts than for the one-day-ahead. Whereas, it is impossible to indicate a better model between the standard GARCH, GJR, EGARCH, and CARR models.

3.3. Robustness check

In this section, a robustness check of the results is performed. We check whether the application of a volatility proxy in place of the true volatility changes the forecasting results. For this purpose, we apply the realized variance. In this study, the realized variance is estimated as the sum of squared 5-min returns which is a common procedure:

$$RV_t = \sum\nolimits_{k=1}^{K} r_{kt}^2, \tag{18}$$

where r_{kt} is a 5-min return, K is the number of intraday observations during a day.

The results of such a comparison are given in the Appendix in Tables A5 and A6, for the MAE and MSE criteria, respectively. In the

Table 5
The out-of-sample evaluation of the one-day-ahead variance forecasts for Monte Carlo simulation: the 90th quantile regression of the loss differential d_t between two models on the lagged real variance.

Parameters of simulated series	GARCH vs CARR		GARCH vs RGARCH		CARR vs RGARCH		GJR vs RGARCH		EGARCH vs RGARCH	
	$\varphi_0(\tau)$	$\varphi_1(\tau)$	$\varphi_0(\tau)$	$\varphi_1(\tau)$	$\varphi_0(\tau)$	$\varphi_1(\tau)$	$\varphi_0(\tau)$	$\varphi_1(\tau)$	$\varphi_0(\tau)$	$\varphi_1(\tau)$
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.045	−0.019	0.038	−0.012	0.014	−0.004	0.036	−0.010	0.035	−0.006
	(0.006)	(0.012)	(0.005)	(0.008)	(0.002)	(0.005)	(0.004)	(0.008)	(0.004	(0.007)
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.337	−0.057	0.279	−0.034	0.103	−0.013	0.269	−0.027	0.272	−0.025
	(0.042)	(0.031)	(0.037)	(0.021)	(0.0135)	(0.013)	(0.03)	(0.02)	(0.02)	(0.01)
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.004	0.000	0.004	0.001	0.002	0.001	0.005	−0.001	0.005	−0.001
	(0.001)	(0.003)	(0.001)	(0.003)	(0.001)	(0.002)	(0.001)	(0.003)	(0.001)	(0.003)
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.035	−0.015	0.030	−0.011	0.011	−0.005	0.031	−0.012	0.031	−0.006
	(0.005)	(0.011)	(0.003)	(0.007)	(0.001)	(0.003)	(0.004)	(0.008)	(0.003)	(0.006)
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.037	−0.001	0.034	−0.000	0.011	0.001	0.034	0.003	0.033	0.004
	(0.004)	(0.011)	(0.006)	(0.012)	(0.002)	(0.004)	(0.005)	(0.009)	(0.005)	(0.011)
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.034	−0.017	0.029	−0.012	0.009	−0.003	0.027	−0.006	0.026	−0.003
	(0.004)	(0.010)	(0.004)	(0.007)	(0.001)	(0.003)	(0.004)	(0.009)	(0.003)	(0.009)
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.044	−0.002	0.004	0.002	0.021	−0.001	0.045	−0.004	0.047	−0.005
	(0.005)	(0.008)	(0.006)	(0.007)	(0.002)	(0.004)	(0.007)	(0.008)	(0.007)	(0.008)

$\varphi_0(\tau)$, $\varphi_1(\tau)$ are parameters of the regression (equation (19)); standard errors are reported in parentheses and are calculated using the Markov chain marginal bootstrap method.

case of the usage of realized variance, the advantage of the RGARCH model is still visible for the MAE measure, but there is no significant difference between the forecasts based on the CARR and RGARCH models for the MSE measure. We can conclude that the application of the volatility proxy has no significant influence on the results.

3.4. Influence of high volatility on the predictive ability

Following Horpestad et al. (2019), we test whether extreme forecast improvements can be explained by the level of market volatility on the previous day. In this regard, lets denote $d_t = (\sigma^2_{A,t} - \sigma^2_{R,t})^2 - (\sigma^2_{B,t} - \sigma^2_{R,t})^2$ as a loss differential between two models M_A and M_B. If d_t is positive, then the forecast based on the model M_B is more precise than the forecast from the model M_A. The loss differential d_t is based on the MSE loss function, but an analogous formula can be presented for the MAE loss function.

The τ-th conditional quantile regression model can be written as:

$$d_t = \varphi_0(\tau) + \varphi_1(\tau)\sigma^2_{R,t-1} + \eta_t(\tau). \tag{19}$$

The results of the parameter estimates for the above quantile regression model are presented in Table 5. We are interested in analyzing high forecast improvements, which is why we use the 90th quantile, however, there are similar results for other high quantiles (e.g. 75th and 95th). For all pairs of models, all estimates of $\varphi_1(\tau)$ are not significantly different from zero at the 10% level. This means that an increase of the true daily variance does not lead to a higher forecast advantage of any of the models.

4. Analysis of currency rates and stock indices

4.1. Data

We also apply the analyzed models on real financial data, i.e. selected six currency rates and nine stock indices. The currency rates are heavily traded currency pairs in the Forex market, namely: AUD/USD, EUR/USD, GBP/USD, NZD/USD, USD/CAD, and USD/JPY. The studied stock indices represent both developed and emerging markets, i.e. Brazil (Bovespa), France (CAC 40), Germany (DAX), the United Kingdom (FTSE 100), Hong Kong (Hang Seng, or HSI), Mexico (IPC), South Korea (KOSPI), Japan (NIKKEI 225), and the United States (S&P 500). The Forex market is open 24 h a day, and thus there is no problem measuring the overnight volatility. The dynamics of the opening jump (the difference between the today's opening price and the yesterday's closing price) is arguably different from the dynamics of the volatility of the trading part of the day. In order to avoid the noise induced by measuring the overnight volatility, we analyze open-to-close returns instead of close-to-close returns for stock indices. It is the standard approach in the realized variance literature.

An evaluation of the considered models is performed for daily data spanning over fourteen years and one quarter, from January 2, 2003, to March 31, 2017. It includes both very volatile periods like the collapse of Lehman Brothers- the worst phase of the global financial crisis, the European sovereign debt crisis, and the Brexit vote but also tranquil periods with low volatility.

The GARCH, GJR, and EGARCH models are formulated based on squared returns. The CARR model is built on ranges, whereas in the RGARCH model, the Parkinson estimator is used. It is vital to compare properties of those volatility proxies and compare them with properties of realized variances, which are used in the paper as true daily variances in a forecasting evaluation.

The descriptive statistics for all considered volatility proxies are presented in Table 6. The percentage of logarithmic returns and ranges, i.e. multiplied by 100, are used in the paper. It is worth noting that the range is not a direct measure of volatility, and it must be

Table 6
Summary statistics of squared returns, range, Parkinson estimator and the realized variance for currency pairs and stock indices.

Assets	Squared returns					Range					Parkinson estimator					Realized variance				
	M	SD	S	EK	LB	M	SD	S	EK	LB	M	SD	S	EK	LB	M	SD	S	EK	LB
Currency pairs																				
AUD/USD	0.71	2.43	15.16	314.00	3 658	1.18	0.74	3.98	31.19	10 408	0.70	1.65	14.34	292.94	7 450	0.77	1.51	11.99	212.11	15 763
EUR/USD	0.38	0.74	5.16	43.95	550	0.93	0.48	1.87	6.55	5 599	0.39	0.51	5.36	46.21	3 781	0.40	0.40	4.92	43.89	11 179
GBP/USD	0.36	1.34	37.49	1 874.86	246	0.88	0.52	5.16	78.57	7 414	0.38	1.12	38.50	1 928.41	705	0.40	0.83	30.09	1 312.88	2 547
NZD/USD	0.72	1.57	9.86	204.12	1 415	1.29	0.69	2.66	13.81	7 349	0.77	1.22	8.83	131.82	5 281	0.91	1.21	8.18	114.35	13 391
USD/CAD	0.36	0.78	6.72	79.70	2 374	0.91	0.48	2.25	10.73	8 791	0.38	0.55	8.44	134.27	6 621	0.42	0.43	4.90	40.45	17 298
USD/JPY	0.42	1.04	10.52	204.89	331	0.95	0.56	3.10	21.01	3 308	0.44	0.84	12.64	249.71	1 066	0.46	0.70	10.98	189.15	3 301
Stock indices																				
BOVESPA	2.83	8.26	14.76	339.92	2 322	2.13	1.26	3.36	21.67	8 376	2.20	4.32	11.29	193.29	8 455	2.24	3.76	8.95	110.43	12 572
CAC 40	1.26	3.22	7.71	85.91	1 405	1.52	0.99	2.31	8.56	10 185	1.18	2.06	5.96	51.24	7 238	1.27	2.22	10.33	174.19	9 580
DAX	1.45	4.16	10.61	174.37	1 553	1.60	1.08	2.47	10.25	11 642	1.34	2.50	6.88	69.33	8 766	1.45	2.62	9.74	158.02	10 202
FTSE 100	0.73	2.04	9.52	146.67	1 684	1.15	0.80	2.75	12.78	12 795	0.71	1.44	8.12	103.83	9 067	0.73	1.37	9.37	141.51	10 884
HSI	0.89	4.14	26.19	845.45	1 635	1.23	0.83	5.67	71.61	4 949	0.79	2.66	27.51	1 028.65	2 018	0.80	1.61	14.43	297.03	5 585
IPC	1.44	4.18	10.93	191.99	1 806	1.42	0.95	2.99	15.19	7 956	1.05	2.15	8.64	114.32	6 639	0.87	1.89	13.20	270.03	4 189
KOSPI	1.09	3.85	19.19	555.41	1 804	1.36	0.97	4.10	34.23	10 369	1.00	2.76	16.89	421.90	6 056	1.01	2.11	12.37	241.01	12 286
NIKKEI 225	1.31	5.28	14.48	268.93	2 307	1.34	0.96	4.09	30.25	6 187	0.98	2.62	12.90	230.32	3 506	1.01	1.83	8.49	99.32	7 727
S&P 500	1.24	4.44	12.54	218.37	3 308	1.26	1.00	3.69	21.96	14 135	0.93	2.52	9.67	120.17	10 652	1.08	2.71	11.52	222.89	11 277

M – mean, SD - standard deviation, S – skewness, EK - excess kurtosis, LB – the Ljung-Box statistic for 10 lags. The sample period is January 2, 2003 to March 31, 2017.

Table 7
The results of the parameter estimates for the GARCH, CARR, RGARCH, GJR, and EGARCH models for currency pairs and stock indices.

Assets	GARCH				CARR			RGARCH			
	γ_0	α_0	α_1	β_1	α_0	α_1	β_1	γ_0	α_0	α_1	β_1
AUD/USD	0.012	0.006	0.061	0.930	0.014	0.127	0.861	0.009	0.006	0.100	0.888
	(0.011)	(0.002)	(0.009)	(0.009)	(0.004)	(0.012)	(0.013)	(0.011)	(0.003)	(0.017)	(0.018)
EUR/USD	0.006	0.001	0.033	0.964	0.005	0.080	0.914	0.002	0.001	0.047	0.949
	(0.009)	(0.001)	(0.004)	(0.005)	(0.002)	(0.008)	(0.008)	(0.009)	(0.001)	(0.009)	(0.009)
GBP/USD	0.003	0.003	0.064	0.930	0.007	0.100	0.892	−0.001	0.004	0.108	0.879
	(0.008)	(0.001)	(0.023)	(0.022)	(0.002)	(0.010)	(0.011)	(0.008)	(0.002)	(0.037)	(0.033)
NZD/USD	0.015	0.010	0.056	0.930	0.024	0.133	0.849	0.009	0.010	0.081	0.900
	(0.013)	(0.003)	(0.009)	(0.011)	(0.007)	(0.015)	(0.019)	(0.012)	(0.004)	(0.019)	(0.023)
USD/CAD	−0.004	0.002	0.049	0.945	0.007	0.099	0.893	−0.003	0.002	0.077	0.913
	(0.008)	(0.001)	(0.006)	(0.006)	(0.002)	(0.008)	(0.009)	(0.008)	(0.001)	(0.010)	(0.012)
USD/JPY	0.008	0.008	0.057	0.926	0.018	0.119	0.862	0.000	0.009	0.115	0.860
	(0.010)	(0.003)	(0.012)	(0.016)	(0.005)	(0.014)	(0.017)	(0.009)	(0.004)	(0.023)	(0.027)
BOVESPA	0.037	0.045	0.063	0.919	0.073	0.172	0.792	0.010	0.044	0.134	0.875
	(0.024)	(0.013)	(0.010)	(0.013)	(0.015)	(0.015)	(0.020)	(0.023)	(0.019)	(0.026)	(0.025)
CAC 40	0.024	0.017	0.089	0.897	0.037	0.203	0.772	−0.011	0.033	0.245	0.746
	(0.014)	(0.007)	(0.019)	(0.022)	(0.008)	(0.017)	(0.020)	(0.014)	(0.01)	(0.037)	(0.036)
DAX	0.041	0.016	0.079	0.909	0.034	0.184	0.794	0.007	0.028	0.187	0.804
	(0.016)	(0.005)	(0.013)	(0.014)	(0.007)	(0.015)	(0.017)	(0.015)	(0.010)	(0.039)	(0.039)
FTSE 100	−0.011	0.006	0.097	0.896	0.025	0.206	0.772	−0.027	0.011	0.248	0.751
	(0.010)	(0.003)	(0.020)	(0.021)	(0.005)	(0.016)	(0.018)	(0.010)	(0.004)	(0.031)	(0.027)
HIS	−0.026	0.013	0.058	0.925	0.026	0.122	0.857	−0.036	0.014	0.089	0.899
	(0.013)	(0.004)	(0.010)	(0.012)	(0.008)	(0.017)	(0.021)	(0.013)	(0.006)	(0.019)	(0.022)
IPC	0.070	0.021	0.084	0.900	0.042	0.169	0.801	0.046	0.018	0.164	0.864
	(0.015)	(0.006)	(0.012)	(0.014)	(0.009)	(0.017)	(0.021)	(0.016)	(0.007)	(0.026)	(0.021)
KOSPI	−0.031	0.011	0.086	0.903	0.025	0.193	0.788	−0.045	0.013	0.183	0.821
	(0.013)	(0.004)	(0.013)	(0.014)	(0.006)	(0.018)	(0.021)	(0.013)	(0.005)	(0.028)	(0.027)
NIKKEI	−0.006	0.049	0.136	0.827	0.053	0.203	0.757	−0.025	0.034	0.289	0.753
225	(0.016)	(0.016)	(0.028)	(0.028)	(0.013)	(0.026)	(0.033)	(0.015)	(0.018)	(0.064)	(0.050)
S&P 500	0.050	0.019	0.102	0.878	0.031	0.200	0.774	0.018	0.016	0.253	0.789
	(0.012)	(0.005)	(0.013)	(0.015)	(0.006)	(0.014)	(0.016)	(0.012)	(0.006)	(0.028)	(0.021)

Assets	GJR					EGARCH				
	γ_0	α_0	α_1	β_1	θ_1	γ_0	α_0	α_1	β_1	θ_1
AUD/USD	0.004	0.006	0.031	0.939	**0.040**	0.001	−0.004	**0.125**	0.990	−0.035
	(0.011)	(0.002)	(0.009)	(0.009)	**(0.012)**	(0.011)	(0.003)	**(0.015)**	(0.003)	(0.009)
EUR/USD	0.001	0.001	0.016	0.969	0.025	0.000	−0.003	0.066	0.995	−0.020
	(0.009)	(0.001)	(0.006)	(0.004)	(0.007)	(0.009)	(0.002)	(0.009)	(0.002)	(0.006)
GBP/USD	0.002	0.003	0.062	0.930	0.004	0.000	−0.009	0.135	0.989	−0.010
	(0.008)	(0.001)	(0.032)	(0.021)	(0.022)	(0.008)	(0.005)	(0.053)	(0.005)	(0.018)
NZD/USD	0.005	0.009	0.017	0.944	0.049	0.002	−0.005	0.113	0.985	−0.031
	(0.012)	(0.003)	(0.010)	(0.012)	(0.012)	(0.012)	(0.002)	(0.017)	(0.004)	(0.010)
USD/CAD	−0.002	0.002	0.054	0.945	−0.012	−0.001	−0.009	0.112	0.991	0.015
	(0.008)	(0.001)	(0.008)	(0.006)	(0.010)	(0.008)	(0.003)	(0.012)	(0.002)	(0.008)
USD/JPY	0.003	0.009	0.041	0.918	0.040	0.001	−0.022	0.147	0.970	−0.037
	(0.010)	(0.003)	(0.010)	(0.017)	(0.020)	(0.010)	(0.008)	(0.020)	(0.009)	(0.016)
BOVESPA	0.004	0.053	0.015	0.921	0.086	−0.006	0.017	0.126	0.983	−0.064
	(0.024)	(0.016)	(0.008)	(0.014)	(0.019)	(0.024)	(0.006)	(0.016)	(0.006)	(0.013)
CAC 40	−0.010	0.019	0.000	0.909	0.148	−0.017	0.001	0.125	0.976	−0.132
	(0.014)	(0.007)	(0.000)	(0.019)	(0.031)	(0.014)	(0.003)	(0.021)	(0.005)	(0.016)
DAX	0.097	0.018	0.004	0.917	0.124	0.004	0.005	0.135	0.977	−0.098
	(0.015)	(0.005)	(0.007)	(0.013)	(0.023)	(0.015)	(0.003)	(0.016)	(0.005)	(0.014)
FTSE 100	−0.029	0.007	0.027	0.906	0.113	−0.033	−0.005	0.163	0.986	0.163
	(0.010)	(0.002)	(0.012)	(0.018)	(0.022)	(0.010)	(0.004)	(0.027)	(0.005)	(0.027)
HIS	−0.035	0.014	0.035	0.926	0.040	−0.038	−0.001	0.135	0.983	−0.034
	(0.013)	(0.004)	(0.013)	(0.012)	(0.017)	(0.013)	(0.003)	(0.023)	(0.005)	(0.014)
IPC	0.043	0.022	0.014	0.909	0.115	0.037	0.006	0.150	0.981	−0.086
	(0.015)	(0.006)	(0.007)	(0.014)	(0.021)	(0.015)	(0.003)	(0.017)	(0.005)	(0.013)
KOSPI	−0.044	0.013	0.042	0.903	0.082	−0.049	−0.001	0.164	0.982	−0.068
	(0.013)	(0.004)	(0.010)	(0.014)	(0.023)	(0.013)	(0.003)	(0.019)	(0.005)	(0.015)
NIKKEI 225	−0.025	0.051	0.070	0.833	0.110	−0.026	0.009	0.232	0.956	−0.083
	(0.015)	(0.016)	(0.037)	(0.025)	(0.048)	(0.014)	(0.008)	(0.040)	(0.013)	(0.030)
S&P 500	0.018	0.020	0.000	0.888	0.175	0.021	−0.003	0.133	0.977	−0.139
	(0.012)	(0.004)	(0.000)	(0.013)	(0.023)	(0.012)	(0.003)	(0.019)	(0.005)	(0.016)

The γ_0 parameter is the constant in the mean equation; α_0, α_1, β_1 are the parameters of the GARCH model (equation (2)), the CARR (equation (8)), and the RGARCH model (equation (12)); α_0 α_1 $\beta_{1,}$ θ_1 are the parameters of the GJR model (equation (4)) and the EGARCH model (equation (5). Standard errors are reported in parentheses. The sample period is January 2, 2003 to March 31, 2017.

Table 8
The in-sample evaluation of the models for currency pairs and stock indices: the logarithm of the likelihood function and the Rivers-Vuong test.

Assets	Log-likelihood					P-value of RV test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH vs CARR	GARCH vs RGARCH	CARR vs RGARCH	GJR vs RGARCH	EGARCH vs RGARCH
AUD/USD	−4031.82	−4041.13	**−4000.49**	−4024.87	−4033.78	0.752	0.000	0.000	0.003	0.000
EUR/USD	−3224.31	−3220.35	−3217.14	**−3216.70**	−3222.03	0.307	0.101	0.260	0.523	0.272
GBP/USD	−3015.01	−2998.86	**−2991.30**	−3014.96	−3017.78	0.048	0.004	0.161	0.004	0.002
NZD/USD	−4368.84	−4350.78	**−4341.66**	−4357.31	−4363.96	0.046	0.000	0.076	0.046	0.007
USD/CAD	−3011.49	−2996.72	**−2981.44**	−3010.49	−3013.89	0.044	0.000	0.004	0.000	0.000
USD/JPY	−3403.01	**−3363.88**	−3367.92	−3396.85	−3392.72	0.003	0.001	0.725	0.007	0.030
BOVESPA	−6374.79	−6368.43	**−6336.22**	−6348.44	−6360.60	0.327	0.000	0.000	0.071	0.001
CAC 40	−5057.37	−5013.05	−4992.55	−4998.09	**−4978.71**	0.003	0.000	0.010	0.339	0.994
DAX	−5210.83	−5176.04	**−5146.19**	−5161.76	−5156.82	0.035	0.000	0.001	0.148	0.242
FTSE 100	−3860.22	−3826.32	**−3783.88**	−3827.44	−3822.85	0.027	0.000	0.000	0.002	0.004
HSI	−3965.24	−3969.07	**−3928.06**	−3960.36	−3963.07	0.602	0.000	0.000	0.000	0.012
IPC	−5173.66	−5198.91	−5152.68	**−5131.87**	−5140.03	0.901	0.029	0.000	0.965	0.852
KOSPI	−4511.62	−4482.24	**−4442.95**	−4494.43	−4481.22	0.037	0.000	0.000	0.000	0.001
NIKKEI 225	−4778.86	−4773.81	**−4729.17**	−4764.31	−4748.93	0.409	0.000	0.007	0.008	0.058
S&P 500	−4523.08	−4551.41	**−4456.87**	−4456.97	−4460.05	0.911	0.000	0.000	0.497	0.413

The highest values of the logarithm of the likelihood function are marked in bold. The p-values of the Rivers-Vuong test are presented for the pairs of models. A p-value lower than the significance level means that the second model is superior to the first model in pair. The sample period is January 2, 2003 to March 31, 2017.

scaled for this purpose (see Sections 2.2).

Squared returns contain substantial noise, and thus their standard deviations are considerably higher than for other volatility proxies. The distributions of all analyzed series exhibit strong skewness and high kurtosis. We can observe that the distribution of realized variances is visibly more similar to the distributions of volatility estimated from the Parkinson estimator than from squared returns. All values for the Ljung-Box statistic show the existence of significant autocorrelation, however, much stronger relations were present for range-based volatility proxies than for squared returns. The presented statistics indicate that the use of range-based measures in volatility models can be useful.

4.2. Modelling volatility

An initial evaluation of the five considered models is performed for the whole range of data. The results of estimation are presented in Table 7. Likewise, in the Monte Carlo simulation the estimates of parameter α_1 are much lower, and the estimates of parameter β_1 are much higher in the standard GARCH model compared with the CARR and RGARCH models. This means that more weight is put on the new information. Such a behavior allows the range-based models to respond faster to changing market conditions. This empirical characteristic has already been reported in the literature (see e.g. Chou et al., 2009; Wu and Liang, 2011; Molnár, 2016; Fiszeder et al., 2019). Parameter θ_1 in the GJR and EGARCH models is responsible for the description of a phenomenon known as the leverage effect, i.e., the negative correlation between volatility and past returns. Estimates of this parameter are statistically significant for most currencies and stock indices.

We compare the five models based on the likelihood function and the Rivers-Vuong test (see Table 8). For eleven out of fifteen assets, the highest values of the likelihood function are for the RGARCH model. It is significantly better (at the 10 % significance level) than the standard GARCH model for all assets except the EUR/USD currency pair and significantly better than the CARR model for all assets except the EUR/USD, GBP/USD, and USD/JPY currency pairs. The RGARCH model is also superior to the GJR and EGARCH models for ten out of fifteen currency pairs and stock indices. On the other hand, the CARR model describes the analyzed series significantly better than the standard GARCH model in eight out of fifteen assets.

4.3. Forecasting volatility

The forecasting performance of the five univariate models is compared in this section. Out-of-sample one-day ahead, five-day-ahead (i.e. for the fifth day ahead), and ten-day-ahead (i.e. for the tenth day ahead) forecasts of variance are formulated based on the GARCH, GJR, EGARCH, CARR, and RGARCH models. The parameters of these models are estimated separately for each day based on a rolling sample of a fixed size 500 (approximately a two-year period; the first in-sample period is from January 2, 2003, to December 31, 2004. The evaluation of forecasts is performed for the period from January 3, 2005, to March 31, 2017. The MSE and MAE measures given in equations (16) and (17) are applied. As the true daily volatility, we use the realized variance calculated as the sum of the squared 5-minute returns, but there are similar conclusions for 15-minute returns. We apply the SPA and MCS tests to analyze whether differences in forecasts are statistically significant. The results of the tests for the one-day-ahead forecasts for the MAE and MSE loss functions are presented in Tables 9 and 10, respectively.

According to the MAE and MSE measures and the SPA and MCS tests, it is not possible to obtain statistically more accurate forecasts

Table 9
The out-of-sample evaluation of the one-day-ahead variance forecasts for currency pairs and stock indices: the realized variance used as a true variance proxy for MAE criterion.

Assets	MAE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
Currency pairs															
AUD/USD	0.299	0.383	**0.236**	0.347	0.345	0.000	0.000	0.598	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
EUR/USD	0.150	0.154	**0.137**	0.173	0.186	0.000	0.000	0.563	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
GBP/USD	0.162	0.160	**0.137**	0.174	0.176	0.000	0.003	0.528	0.001	0.000	0.000	0.001	1.000*	0.000	0.000
NZD/USD	0.355	0.345	**0.323**	0.369	0.370	0.000	0.000	0.606	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
USD/CAD	0.140	0.140	**0.123**	0.153	0.161	0.000	0.000	0.532	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
USD/JPY	0.220	0.227	**0.190**	0.254	0.243	0.000	0.000	0.518	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
Stock indices															
BOVESPA	1.351	1.490	**1.224**	1.266	**1.224**	0.000	0.000	0.729	0.168	0.675	0.003	0.000	1.000*	0.511*	0.991*
CAC 40	0.654	0.631	**0.588**	0.669	0.633	0.000	0.000	0.591	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
DAX	0.693	0.733	**0.609**	0.672	0.640	0.000	0.000	0.959	0.001	0.041	0.000	0.000	1.000*	0.010	0.088
FTSE 100	0.373	0.367	**0.330**	0.359	0.349	0.000	0.000	0.497	0.001	0.026	0.000	0.000	1.000*	0.005	0.031
HSI	0.553	0.545	**0.463**	0.548	0.505	0.003	0.000	0.611	0.005	0.000	0.000	0.000	1.000*	0.000	0.000
IPC	0.904	1.008	0.890	**0.769**	0.803	0.000	0.000	0.000	0.503	0.001	0.000	0.000	0.000	1.000*	0.002
KOSPI	0.483	0.522	**0.431**	0.505	0.479	0.001	0.000	0.492	0.004	0.024	0.005	0.000	1.000*	0.003	0.017
NIKKEI 225	0.835	0.881	**0.684**	0.792	0.698	0.002	0.000	0.779	0.006	0.221	0.000	0.000	1.000*	0.007	0.419*
S&P 500	0.708	0.895	0.651	0.665	**0.601**	0.000	0.000	0.033	0.003	0.528	0.000	0.000	0.024	0.005	1.000*

The realized variance is estimated as the sum of the squared 5-minute returns. The lowest values of the MAE measure are in bold; p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing which model is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly; * indicates that model belongs to the model confidence set with a confidence level of 0.90. The evaluation period is January 3, 2005 to March 31, 2017.

Table 10
The out-of-sample evaluation of the one-day-ahead variance forecasts for currency pairs and stock indices: the realized variance used as a variance proxy for the MSE criterion.

Assets	MSE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
Currency pairs															
AUD/USD	0.809	1.737	**0.383**	1.145	1.005	0.019	0.006	0.959	0.036	0.058	0.026	0.008	1.000*	0.021	0.026
EUR/USD	0.088	0.095	**0.074**	0.103	0.107	0.002	0.002	0.661	0.000	0.000	0.003	0.002	1.000*	0.001	0.000
GBP/USD	0.500	0.661	**0.336**	0.819	0.769	0.052	0.519	0.910	0.205	0.258	0.342*	0.342*	1.000*	0.342*	0.342*
NZD/USD	0.914	0.798	**0.705**	0.970	0.965	0.049	0.030	0.963	0.007	0.046	0.032	0.032	1.000*	0.002	0.025
USD/CAD	0.079	0.086	**0.059**	0.090	0.099	0.000	0.001	0.549	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
USD/JPY	0.376	0.427	**0.269**	0.455	0.429	0.006	0.019	0.590	0.006	0.016	0.013	0.013	1.000*	0.012	0.013
Stock indices															
BOVESPA	9.638	11.797	7.223	9.524	**6.753**	0.015	0.016	0.375	0.096	0.917	0.038	0.011	0.382*	0.105*	1.000*
CAC 40	3.105	2.784	**2.739**	3.453	2.877	0.030	0.527	0.909	0.007	0.202	0.078	0.747*	1.000*	0.007	0.376*
DAX	4.314	4.311	**3.152**	4.246	3.857	0.050	0.020	0.905	0.086	0.270	0.028	0.028	1.000*	0.028	0.256*
FTSE 100	1.128	1.074	**0.953**	1.127	1.008	0.019	0.041	0.868	0.018	0.284	0.070	0.037	1.000*	0.070	0.482*
HSI	4.320	3.830	**2.695**	4.356	2.910	0.028	0.170	0.979	0.048	0.336	0.013	0.103*	1.000*	0.026	0.312*
IPC	4.085	4.466	4.060	3.215	**3.045**	0.011	0.020	0.032	0.062	0.945	0.006	0.005	0.010	0.075	1.000*
KOSPI	**1.748**	4.142	2.229	2.044	1.956	0.977	0.108	0.535	0.392	0.598	1.000*	0.214*	0.302*	0.536*	0.600*
NIKKEI 225	7.929	7.029	**3.736**	6.696	3.794	0.044	0.025	0.540	0.024	0.461	0.007	0.008	1.000*	0.020	0.885*
S&P 500	4.363	8.105	4.025	4.396	**3.190**	0.011	0.010	0.049	0.034	0.630	0.012	0.001	0.038	0.015	1.000*

The realized variance is estimated as the sum of the squared 5-minute returns. The lowest values of the MSE measure are in bold; p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing which model is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly; * indicates that model belongs to the model confidence set with a confidence level of 0.90. The evaluation period is January 3, 2005 to March 31, 2017.

Table 11
The out-of-sample evaluation of the one-day-ahead variance forecasts for currency pairs and stock indices: the 90th quantile regression of the loss differential d_t between two models on the lagged variance proxy.

Assets	GARCH vs CARR		GARCH vs RGARCH		CARR vs RGARCH		GJR vs RGARCH		EGARCH vs RGARCH	
	$\varphi_0(\tau)$	$\varphi_1(\tau)$	$\varphi_0(\tau)$	$\varphi_1(\tau)$	$\varphi_0(\tau)$	$\varphi_1(\tau)$	$\varphi_0(\tau)$	$\varphi_1(\tau)$	$\varphi_0(\tau)$	$\varphi_1(\tau)$
AUD/USD	−0.054	0.349	−0.301	1.613	−1.454	7.477	−0.001	0.203	0.012	0.192
	(0.007)	(0.020)	(0.044)	(0.124)	(0.203)	(0.598)	(0.003)	(0.009)	(0.005)	(0.015)
EUR/USD	−0.008	0.121	−0.008	0.112	−0.014	0.173	−0.708	3.772	−0.275	1.632
	(0.001)	(0.007)	(0.001)	(0.005)	(0.001)	(0.006)	(0.102)	(0.273)	(0.042)	(0.099)
GBP/USD	−0.048	0.476	−0.023	0.247	−0.037	0.378	−0.058	0.620	−0.020	0.293
	(0.019)	(0.105)	(0.003)	(0.017)	(0.003)	(0.014)	(0.023)	(0.108)	(0.004)	(0.017)
NZD/USD	−0.163	0.692	−0.151	0.646	−0.122	0.529	−0.239	0.997	−0.207	0.894
	(0.022)	(0.046)	(0.027)	(0.065)	(0.014)	(0.027)	(0.032)	(0.063)	(0.037)	(0.093)
USD/CAD	−0.012	0.134	−0.016	0.169	−0.018	0.185	−0.023	0.268	−0.025	0.299
	(0.001)	(0.005)	(0.002)	(0.009)	(0.002)	(0.008)	(0.002)	(0.011)	(0.002)	(0.010)
USD/JPY	−0.016	0.230	−0.023	0.302	−0.029	0.335	−0.055	0.704	−0.022	0.408
	(0.004)	(0.014)	(0.004)	(0.015)	(0.004)	(0.016)	(0.014)	(0.063)	(0.004)	(0.013)
BOVESPA	−0.827	2.005	−1.811	3.582	−5.343	9.579	−1.853	3.811	−0.545	2.057
	(0.164)	(0.1313)	(0.207)	(0.162)	(0.690)	(0.775)	(0.228)	(0.146)	(0.370)	(0.261)
CAC 40	−0.328	1.694	−0.326	1.709	−0.250	1.118	−0.457	2.253	−0.190	1.210
	(0.037)	(0.072)	(0.038)	(0.097)	(0.032)	(0.081)	(0.044)	(0.069)	(0.046)	(0.069)
DAX	−0.212	1.242	−0.393	1.916	−0.563	2.418	−0.382	1.887	−0.286	1.502
	(0.038)	(0.065)	(0.049)	(0.096)	(0.054)	(0.094)	(0.060)	(0.116)	(0.046)	(0.062)
FTSE 100	−0.080	0.800	−0.098	0.961	−0.093	0.787	−0.104	0.989	−0.084	0.848
	(0.006)	(0.016)	(0.009)	(0.033)	(0.011)	(0.040)	(0.011)	(0.040)	(0.011)	(0.040)
HSI	−0.515	2.622	−0.591	2.948	−0.314	1.688	−0.530	2.728	−0.185	1.118
	(0.023)	(0.042)	(0.039)	(0.117)	(0.045)	(0.100)	(0.024)	(0.036)	(0.054)	(0.141)
IPC	−0.199	1.418	−0.248	1.676	−0.305	2.192	−0.232	1.782	−0.120	1.608
	(0.026)	(0.064)	(0.022)	(0.042)	(0.024)	(0.053)	(0.032)	(0.064)	(0.037)	(0.051)
KOSPI	−0.176	1.339	−0.195	1.412	−0.547	3.346	−0.447	3.007	−0.355	2.455
	(0.025)	(0.076)	(0.022)	(0.063)	(0.089)	(0.248)	(0.076)	(0.203)	(0.039)	(0.093)
NIKKEI 225	0.444	2.757	−1.060	5.837	−2.138	11.053	−1.034	5.793	−0.543	3.226
	(0.066)	(0.136)	(0.137)	(0.291)	(0.334)	(0.906)	(0.174)	(0.550)	(0.068)	(0.171)
S&P 500	−0.184	1.693	−0.175	1.722	−1.782	13.893	−0.201	2.058	−0.153	1.599
	(0.025)	(0.063)	(0.024)	(0.061)	(0.244)	(0.555)	(0.032)	(0.108)	(0.029)	(0.092)

The realized variance used as the true daily variance and estimated as the sum of squared 5-minute returns; $\varphi_0(\tau)$, $\varphi_0(\tau)$ are parameters of the regression (equation (19)); standard errors are reported in parentheses and are calculated using the Markov chain marginal bootstrap method. The evaluation period is January 3, 2005 to March 31, 2017.

of the variance than from the RGARCH model for all currency pairs and stock indices, except for the IPC and S&P 500 indices. On the other hand, it is difficult to indicate the second-best model when considering the GARCH, GJR, EGARCH, and CARR models.

The results for the five-day-ahead forecasts are presented in the Appendix in Tables A7 and A8 for the MAE and MSE criteria, respectively, and the results for the ten-day-ahead forecasts are given in Tables A9 and A10 for the MAE and MSE measures, respectively. The advantage of the RGARCH model is rather minor for the five-day-ahead forecasts, and there is almost no difference between the analyzed models for the ten-day-ahead forecasts. For some of the tested time series, there is an indication favoring the RGARCH model, but it is not conclusive.

4.4. *Influence of market volatility on the predictive ability*

Similarly as for the data from to the Monte Carlo simulations, we test whether extreme forecast improvements can be explained by the level of market volatility in the previous day. Table 11 presents results for 90th conditional quantile regression model (equation (19)) for selected pairs of models. Comparable results were obtained for other high quantiles, the 75th and 95th quantile.

All estimates of $\varphi_1(\tau)$ are positive and significantly different from zero at the 10 % level.

This means that for large improvements of variance forecasts, an increase of the realized variance leads to the higher forecast advantage of the RGARCH model over the GARCH, GJR, EGARCH, and CARR models. Likewise, an increase of market volatility leads to a rise in forecasting predominance of the CARR model over the GARCH model. This may explain why in other studies this model performs better than the GARCH model (see Chou, 2005; Chou and Wang, 2007; Liu and Wu, 2017; Fiszeder and Fałdziński, 2019). The presented results for the loss differentials are based on the MSE loss function, however, results for the MAE loss function are very similar. It is worth noting that these results for financial series are different than results for the simulated series presented in Section 3.4. It seems that the behavior of prices on real financial markets during turbulent periods have different properties than simulated prices. This indicates that the dynamics of volatility in the real world differs from the dynamics of volatility in our simulation.

5. Conclusions

Due to availability of daily high and low prices, volatility models which utilize the high-low range, have emerged and become

Table A1
The out-of-sample evaluation of the five-day-ahead variance forecasts for Monte Carlo simulation: the real variance used for the MAE criterion.

Parameters of simulated series	MAE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.179	0.186	**0.169**	0.180	0.175	0.000	0.000	0.496	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.487	0.481	**0.459**	0.488	0.475	0.000	0.000	0.490	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.066	0.076	**0.062**	0.066	0.064	0.000	0.000	0.512	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.161	0.159	**0.152**	0.161	0.167	0.000	0.000	0.552	0.000	0.033	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.175	0.181	**0.163**	0.175	0.170	0.000	0.000	0.507	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.090	0.092	**0.071**	0.090	0.097	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.214	0.220	**0.203**	0.214	0.207	0.000	0.000	0.966	0.000	0.034	0.000	0.000	1.000*	0.000	0.054

The lowest values of the MAE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90.

Table A2
The out-of-sample evaluation of the five-day-ahead variance forecasts for Monte Carlo simulation: the real variance used for the MSE criterion.

Parameters of simulated series	MSE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.070	0.066	**0.063**	0.070	0.075	0.000	0.000	0.528	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.512	0.470	**0.465**	0.513	0.555	0.000	0.033	0.967	0.000	0.000	0.000	0.059	1.000*	0.000	0.000
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.012	0.012	**0.011**	0.012	0.013	0.000	0.000	0.545	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.053	0.052	**0.049**	0.053	0.164	0.000	0.000	0.558	0.000	0.069	0.038	0.038	1.000*	0.038	0.038
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.084	0.077	**0.073**	0.084	0.091	0.000	0.000	0.568	0.000	0.001	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.015	0.014	**0.010**	0.016	0.020	0.000	0.000	0.549	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.147	0.137	**0.134**	0.146	0.157	0.001	0.000	0.683	0.001	0.005	0.001	0.002	1.000*	0.001	0.002

The lowest values of the MSE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90.

Table A3
The out-of-sample evaluation of the ten-day-ahead variance forecasts for Monte Carlo simulation: the real variance used for the MAE criterion.

Parameters of simulated series	MAE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.184	0.243	**0.173**	0.184	0.185	0.000	0.000	0.502	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.499	0.534	**0.472**	0.500	0.503	0.000	0.000	0.505	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.066	0.076	**0.063**	0.066	0.066	0.000	0.000	0.504	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.163	0.225	**0.154**	0.163	0.175	0.000	0.000	0.540	0.000	0.002	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.175	0.235	**0.164**	0.175	0.177	0.000	0.000	0.511	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.163	0.243	**0.154**	0.163	0.166	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.213	0.258	**0.204**	0.213	0.208	0.000	0.000	0.506	0.000	0.006	0.000	0.000	1.000*	0.000	0.007

The lowest values of the MAE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90.

Table A4
The out-of-sample evaluation of the ten-day-ahead variance forecasts for Monte Carlo simulation: real variance used for the MSE criterion.

Parameters of simulated series	MSE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.079	0.100	**0.073**	0.079	0.097	0.000	0.000	0.514	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.582	0.569	**0.536**	0.583	0.711	0.000	0.000	0.550	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.012	0.025	**0.010**	0.012	0.014	0.000	0.000	0.531	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.058	0.081	**0.053**	0.058	0.191	0.000	0.000	0.547	0.000	0.026	0.003	0.003	1.000*	0.003	0.003
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.082	0.101	**0.073**	0.082	0.098	0.000	0.000	0.533	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.057	0.091	**0.053**	0.058	0.070	0.000	0.000	0.517	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.149	0.157	**0.138**	0.149	0.171	0.000	0.000	0.554	0.000	0.000	0.000	0.000	1.000*	0.000	0.000

The lowest values of the MSE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90.

Table A5
The out-of-sample evaluation of the one-day-ahead variance forecasts for Monte Carlo simulation: the realized variance used as a true variance proxy for MAE criterion.

Parameters of simulated series	MAE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.0937	0.071	**0.069**	0.094	0.094	0.000	0.028	0.497	0.000	0.000	0.000	0.057	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.254	0.192	**0.188**	0.255	0.255	0.000	0.0278	0.973	0.000	0.000	0.000	0.059	1.000*	0.000	0.000
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.034	**0.025**	**0.025**	0.034	0.035	0.000	0.350	0.651	0.000	0.000	0.000	0.690*	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.084	0.063	**0.062**	0.084	0.094	0.000	0.051	0.950	0.000	0.005	0.000	0.108*	1.000*	0.000	0.004
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.096	0.071	**0.070**	0.097	0.098	0.000	0.084	0.916	0.000	0.000	0.000	0.160*	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.085	**0.064**	**0.064**	0.085	0.085	0.000	0.512	0.488	0.000	0.000	0.000	1.000*	0.975*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.098	0.073	**0.069**	0.099	0.101	0.000	0.002	0.501	0.000	0.000	0.000	0.002	1.000*	0.000	0.000

The realized variance is estimated as the sum of squared intraday returns. The lowest values of the MAE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90.

Table A6
The out-of-sample evaluation of the one-day-ahead variance forecasts for Monte Carlo simulation: the realized variance used as a true variance proxy for MSE criterion.

Parameters of simulated series	MSE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.985$	0.019	**0.010**	**0.010**	0.019	0.019	0.000	0.628	0.372	0.000	0.000	0.000	1.000*	0.746*	0.000	0.000
$\ln\overline{\sigma} = -2.0, \beta = 0.75, \rho_H = 0.985$	0.136	**0.072**	0.723	0.138	0.140	0.000	0.631	0.369	0.000	0.000	0.000	1.000*	0.741*	0.000	0.000
$\ln\overline{\sigma} = -3.0, \beta = 0.75, \rho_H = 0.985$	0.013	**0.011**	**0.011**	0.003	0.003	0.010	0.703	0.297	0.000	0.000	0.008	1.000*	0.587*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.7, \rho_H = 0.985$	0.014	**0.008**	**0.008**	0.014	0.111	0.000	0.780	0.773	0.000	0.076	0.080	1.000*	0.999*	0.080	0.080
$\ln\overline{\sigma} = -2.5, \beta = 0.8, \rho_H = 0.985$	0.020	0.011	**0.010**	0.020	0.022	0.000	0.162	0.838	0.000	0.000	0.000	0.317*	1.000*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.98$	0.014	**0.008**	**0.008**	0.014	0.015	0.000	0.501	0.027	0.000	0.000	0.000	1.000*	0.049*	0.000	0.000
$\ln\overline{\sigma} = -2.5, \beta = 0.75, \rho_H = 0.99$	0.022	**0.012**	**0.012**	0.023	0.025	0.000	0.154	0846	0.000	0.000	0.000	0.282*	1.000*	0.000	0.000

The realized variance is estimated as the sum of squared intraday returns. The lowest values of the MSE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90.

Table A7
The out-of-sample evaluation of the five-day-ahead variance forecasts for currency pairs and stock indices: the realized variance used as a true variance proxy for the MAE criterion.

Assets	MAE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
AUD/USD	0.373	0.432	**0.369**	0.382	0.370	0.608	0.000	0.705	0.319	0.820	0.974*	0.092	1.000*	0.778*	0.974*
EUR/USD	0.164	0.176	**0.159**	0.178	0.190	0.007	0.000	0.510	0.000	0.000	0.007	0.000	1.000*	0.000	0.000
GBP/USD	0.177	0.177	**0.171**	0.172	**0.171**	0.269	0.251	0.835	0.771	0.861	0.729*	0.592*	0.955*	0.955*	1.000*
NZD/USD	0.394	0.395	0.434	**0.382**	0.390	0.028	0.096	0.000	0.964	0.037	0.008	0.014	0.000	1.000*	0.027
USD/CAD	0.156	**0.153**	0.162	0.159	0.167	0.246	0.870	0.027	0.005	0.000	0.236*	1.000*	0.040	0.077	0.001
USD/JPY	0.256	0.290	**0.244**	0.268	0.2650	0.011	0.000	0.528	0.000	0.000	0.004	0.000	1.000*	0.000	0.000
BOVESPA	1.431	1.542	**1.175**	1.322	1.323	0.000	0.000	0.506	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
CAC 40	0.705	0.711	**0.673**	0.708	0.713	0.065	0.053	0.689	0.034	0.004	0.035	0.022	1.000*	0.022	0.009
DAX	0.741	0.785	**0.680**	0.729	0.711	0.035	0.013	0.979	0.008	0.060	0.014	0.005	1.000*	0.007*	0.040
FTSE 100	0.427	0.427	**0.377**	0.390	0.387	0.007	0.001	0.896	0.141	0.259	0.072	0.010	1.000*	0.269*	0.288*
HSI	0.553	0.545	**0.463**	0.548	0.505	0.003	0.000	0.611	0.005	0.000	0.000	0.000	1.000*	0.000	0.000
IPC	0.904	1.008	0.890	**0.769**	0.803	0.000	0.000	0.000	0.503	0.001	0.000	0.000	0.000	1.000*	0.002
KOSPI	0.567	0.615	**0.485**	0.542	0.554	0.000	0.000	0.619	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
NIKKEI 225	0.955	0.993	**0.643**	0.854	0.751	0.000	0.000	0.679	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
S&P 500	0.768	0.950	**0.644**	0.709	0.671	0.000	0.000	0.940	0.000	0.095	0.000	0.000	1.000*	0.001	0.115*

The realized variance is estimated as the sum of squared of 5-minute returns. The lowest values of the MAE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90. The evaluation period is 3 January 2005 to 31 March 2017.

Table A8

The out-of-sample evaluation of the five-day-ahead variance forecasts for currency pairs and stock indices: the realized variance used as a true variance proxy for the MSE criterion.

Assets	MSE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
AUD/USD	1.721	2.158	**1.393**	1.538	1.306	0.078	0.014	0.464	0.237	0.954	0.122*	0.052	0.552*	0.179*	1.000*
EUR/USD	0.105	0.108	**0.104**	0.107	0.112	0.648	0.323	0.849	0.419	0.009	0.628*	0.628*	1.000*	0.628*	0.389*
GBP/USD	0.763	0.676	0.684	0.670	**0.665**	0.106	0.520	0.340	0.323	0.965	0.191*	0.358*	0.332*	0.358*	1.000*
NZD/USD	1.131	**1.051**	1.293	1.063	1.117	0.113	0.754	0.001	0.598	0.017	0.224*	1.000*	0.008	0.809*	0.398*
USD/CAD	**0.101**	**0.101**	0.103	**0.101**	0.110	0.875	0.591	0.504	0.895	0.007	0.999*	0.999*	0.923*	1.000*	0.205*
USD/JPY	0.500	0.510	0.494	**0.482**	0.486	0.356	0.190	0.331	0.909	0.680	0.384*	0.298*	0.549*	1.000*	0.663*
BOVESPA	10.566	11.822	**9.155**	9.438	8.884	0.086	0.018	0.471	0.442	0.938	0.282*	0.199*	0.590*	0.590*	1.000*
CAC 40	3.585	**3.494**	3.922	3.761	3.801	0.675	0.838	0.166	0.274	0.295	0.693*	1.000*	0.420*	0.609*	0.609*
DAX	5.030	5.666	**4.588**	4.738	4.826	0.561	0.223	0.906	0.734	0.522	0.706*	0.545*	1.000*	0.739*	0.739*
FTSE 100	1.336	1.346	1.287	1.281	**1.213**	0.137	0.265	0.232	0.065	0.998	0.080	0.133*	0.225*	0.133*	1.000*
HIS	4.320	3.830	**2.695**	4.356	2.910	0.028	0.170	0.979	0.048	0.336	0.013	0.103*	1.000*	0.026	0.312*
IPC	4.085	4.466	4.060	3.215	**3.045**	0.011	0.020	0.032	0.062	0.945	0.006	0.005	0.010	0.075	1.000*
KOSPI	2.770	3.262	2.476	**2.418**	2.992	0.151	0.153	0.731	0.974	0.209	0.116*	0.037	0.813*	1.000*	0.116*
NIKKEI 225	11.670	9.838	**3.370**	6.386	4.216	0.039	0.016	0.928	0.006	0.073	0.013	0.011	1.000*	0.013	0.090
S&P 500	5.169	8.537	4.473	4.696	**4.348**	0.056	0.016	0.556	0.365	0.818	0.179*	0.011	0.682*	0.604*	1.000*

The realized variance is estimated as the sum of squared of 5-minute returns. The lowest values of the MSE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90. The evaluation period is 3 January 2005 to 31 March 2017.

Table A9
The out-of-sample evaluation of the ten-day-ahead variance forecasts for currency pairs and stock indices: the realized variance used as true variance proxy for the MAE criterion.

Assets	MAE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
AUD/USD	0.431	0.535	0.448	0.429	**0.396**	0.131	0.000	0.067	0.059	0.974	0.056	0.001	0.045	0.045	1.000*
EUR/USD	**0.172**	0.203	0.173	0.187	**0.172**	0.608	0.000	0.392	0.000	0.000	1.000*	0.000	0.759*	0.000	1.000*
GBP/USD	0.192	0.202	0.189	**0.173**	**0.173**	0.093	0.000	0.001	0.910	0.586	0.040	0.000	0.005	1.000*	0.798*
NZD/USD	0.402	0.426	0.497	**0.400**	0.402	0.394	0.002	0.000	0.897	0.401	0.580*	0.005	0.000	1.000*	0.580*
USD/CAD	**0.162**	0.170	0.186	0.165	0.172	0.664	0.002	0.000	0.008	0.000	1.000*	0.000	0.000	0.003	0.000
USD/JPY	0.271	0.342	**0.259**	0.278	0.266	0.087	0.000	0.996	0.000	0.080	0.031	0.000	1.000*	0.003	0.080
BOVESPA	1.510	1.597	**1.222**	1.407	1.409	0.000	0.000	0.512	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
CAC 40	0.742	0.760	0.789	**0.727**	0.730	0.304	0.047	0.001	0.764	0.623	0.573*	0.191*	0.014	1.000*	0.808*
DAX	0.805	0.885	0.840	**0.773**	0.751	0.097	0.000	0.000	0.064	0.992	0.059	0.000	0.000	0.059	1.000*
FTSE 100	0.450	0.504	0.429	**0.424**	0.414	0.004	0.000	0.166	0.100	0.983	0.001	0.000	0.084	0.084	1.000*
HSI	0.659	0.666	**0.475**	0.656	0.570	0.003	0.000	0.999	0.004	0.001	0.000	0.000	1.000*	0.000	0.000
IPC	0.973	1.054	**0.689**	0.818	0.832	0.000	0.000	0.547	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
KOSPI	0.589	0.665	**0.564**	0.582	0.598	0.168	0.000	0.922	0.306	0.018	0.199	0.003	1.000*	0.239	0.082
NIKKEI 225	1.109	1.037	**0.656**	0.873	0.758	0.001	0.000	0.687	0.000	0.000	0.000	0.000	1.000*	0.000	0.000
S&P 500	0.792	0.926	**0.674**	0.749	0.713	0.000	0.000	0.571	0.002	0.007	0.000	0.000	1.000*	0.000	0.005

The realized variance is estimated as the sum of squared of 5-minute returns. The lowest values of the MAE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90. The evaluation period is 3 January 2005 to 31 March 2017.

Table A10
The out-of-sample evaluation of the ten-day-ahead variance forecasts for currency pairs and stock indices: the realized variance used as a true variance proxy for the MSE criterion.

Assets	MSE					P-value of SPA test					P-value of MCS test				
	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH	GARCH	CARR	RGARCH	GJR	EGARCH
AUD/USD	2.710	2.679	3.127	2.319	**1.733**	0.031	0.013	0.041	0.017	0.626	0.011	0.011	0.011	0.011	1.000*
EUR/USD	**0.114**	0.126	0.122	0.116	**0.114**	0.995	0.008	0.097	0.286	0.067	1.000*	0.001	0.033	0.152*	1.000*
GBP/USD	0.900	0.706	0.720	**0.673**	0.676	0.047	0.050	0.015	0.996	0.395	0.018	0.018	0.018	1.000*	0.388*
NZD/USD	1.189	**1.112**	1.534	1.184	1.243	0.223	0.927	0.000	0.253	0.061	0.195*	1.000*	0.000	0.195*	0.195*
USD/CAD	0.110	0.112	0.131	**0.109**	0.119	0.568	0.398	0.027	0.995	0.006	0.545*	0.545*	0.000	1.000*	0.098*
USD/JPY	0.560	0.587	0.539	**0.493**	0.513	0.113	0.025	0.008	0.974	0.257	0.025	0.010	0.114*	1.000*	0.279*
BOVESPA	12.560	13.725	11.822	12.079	**11.649**	0.210	0.087	0.523	0.525	0.978	0.196*	0.196*	0.614*	0.333*	1.000*
CAC 40	**3.691**	3.797	4.782	3.955	4.047	0.949	0.000	0.402	0.117	0.111	1.000*	0.403*	0.012	0.168	0.008
DAX	5.822	7.430	6.311	**5.454**	5.514	0.394	0.044	0.011	0.940	0.569	0.633*	0.175*	0.175*	1.000*	0.739*
FTSE 100	1.632	1.722	1.640	1.486	**1.371**	0.061	0.050	0.027	0.003	0.852	0.008	0.001	0.001	0.008	1.000*
HIS	7.253	4.184	**2.940**	7.434	4.052	0.034	0.063	0.958	0.040	0.068	0.015	0.024	1.000*	0.020	0.024
IPC	4.954	5.008	3.965	3.631	**3.503**	0.020	0.002	0.095	0.091	0.983	0.009	0.005	0.073	0.073	1.000*
KOSPI	3.337	**3.076**	4.178	3.300	3.841	0.109	0.953	0.022	0.123	0.039	0.084	1.000*	0.019	0.084	0.056
NIKKEI 225	19.539	11.711	**3.783**	6.760	4.290	0.016	0.017	0.830	0.033	0.299	0.005	0.014	1.000*	0.024	0.342*
S&P 500	5.100	5.481	**4.782**	5.211	4.795	0.500	0.265	0.885	0.220	0.737	0.509*	0.420*	1.000*	0.420*	0.949*

The realized variance is estimated as the sum of squared of 5-minute returns. The lowest values of the MSE measure are in bold, p-values for the SPA and MCS tests are computed by the bootstrapping methodology (Hansen, 2005). We apply the SPA test five times, each time changing the model which is the benchmark. It means that the given SPA p-value refers to the benchmark model specified in the heading of the table. The MCS test is performed for the five models jointly, * indicates that model belongs to the model confidence set with a confidence level of 0.90. The evaluation period is 3 January 2005 to 31 March 2017.

increasingly popular. Existing literature shows that the range-based models outperform the standard GARCH models (see e.g. Chou, 2005; Brandt and Jones, 2006; Asai, 2013; Fiszeder and Perczak, 2016; Molnár, 2016; Fiszeder and Fałdziński, 2019; Fiszeder et al., 2019; Xie, 2019). However, a comparison of the performance of range-based volatility models is missing in the literature. This paper fills that gap by comparing five models, three based on closing prices, i.e., the GARCH, GJR, EGARCH models, and two incorporating high and low prices, namely the CARR model and the Range-GARCH model. The CARR model describes the dynamics of the conditional mean of the price range, while the Range-GARCH model is similar to the standard GARCH model, but instead of the squared returns, it utilizes a more efficient volatility estimator based on the daily range.

We evaluate the competing models based on Monte Carlo experiments. For a simulated time series, the range-based models definitely win over the standard GARCH model and two asymmetric models, while the performance of the RGARCH model and the CARR model is similar. It is also crucial to compare models using real-world financial data. We therefore utilize two data sets: six currencies pairs and nine stocks indices. We find that the Range-GARCH model outperforms the GARCH, GJR, EGARCH, and CARR models both in in-sample and out-of-sample analysis. On the other hand, the CARR model performs sometimes better, but sometimes worse than the standard GARCH model and two asymmetric models. It means that the way in which low and high prices are used in the model is essential.

Additionally, we test whether extreme forecast improvements can be explained by the level of volatility. We show that an increase of the realized variance leads to a higher forecast advantage of the CARR model over the GARCH model, and the predominance of the RGARCH model over the standard GARCH, GJR, EGARCH, and CARR models for empirical time series. Altogether, we conclude that range-based models are better than the standard GARCH model, and when considering these two range-based models, Range-GARCH is preferred.

The study can be extended in the future to other variants of the GARCH models which describe other properties of financial time series like long memory or heavy tails of conditional distributions. Modelling such features can improve forecasts, but this applies to all models considered in this paper. It means that not only can the standard GARCH model be extended for this purpose, but so can the CARR and RGARCH models, as the GARCH model has many various extensions. We believe this as an interesting topic for further studies.

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CRediT authorship contribution statement

Marcin Fałdziński: Conceptualization, Methodology, Data curation, Formal analysis, Writing – original draft, Writing – review & editing. **Piotr Fiszeder:** Conceptualization, Methodology, Data curation, Formal analysis, Writing – original draft, Writing – review & editing. **Peter Molnár:** Conceptualization, Methodology, Writing – original draft, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

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